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AD312699
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NRL ltr dtd 29 Jun 98; NRL, 29 Jun 1998

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DATED - JULY 1972

DOD 5000.1R & EXECUTIVE ORDER 11652
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CONTENTS

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ABSTRACT
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The characteristics of the various factors involved in the reception of ~~very-low-frequency~~ (vlf) radio signals with rectangular open-core loop antennas in air and water ~~have been~~ were considered with regard to the prediction of the maximum permissible range capability for specified minimal satisfactory message-reception conditions. A mathematical expression, termed the "system equation," ~~has been~~ was devised relating such factors as radiated power, propagation attenuation loss, voltage- and field-interface losses, depth-of-submergence loss, and loop-antenna pickup capability gain with operation in water, to the voltage induced in a loop antenna in air. The variation of these factors as functions of frequency, water conductivity, and loop-antenna dimensions ~~has been~~ was accurately calculated with the aid of the U.S. Naval Research Laboratory High-Speed Electronic Digital Computer (NAREC). It ~~has been~~ was determined that for a fixed field strength in the air, the signal voltage induced in a submerged loop antenna will increase with a frequency increase at shallow depths of loop submergence but will eventually decrease with frequency increase as the loop submergence increases. Relative air-to-water performance data ~~has been~~ were employed in typical calculations of communication-system range based on the approximate sensitivity of the omnidirectional-loop system currently in use aboard most U.S. submarines. Many of the basic equations employed are expansions of original work by the late Dr. O. Norgorden of this Laboratory.

It has been shown that normally, in communication-system range computations, specifying the overall receiving-system field sensitivity in air automatically includes both the dimensional and electrical loop-antenna-system design parameters for air operation, leaving, in addition, the voltage-interface loss effect (which in turn is a function of the loop dimensions) to be considered along with the loop depth-of-submergence loss for underwater operation. Expressing receiving-system performance in terms of the variations to be expected in basic system parameters with respect to a fixed voltage sensitivity is, however, more indicative of realizable system performance capability rather than to a fixed field sensitivity.

It has been shown that with respect to a reference radio field in air, the signal voltage induced in a submerged loop antenna may increase as the loop-antenna dimensions are increased, but that this improvement is subject in a very real sense to practical limitations imposed by the need for optimum loop-system electrical design and operational factors which are a function of the water environment in which the submarine operates. A determination of the optimum overall loop-antenna design for submerged radio reception must consider the mutual optimization of all the loop-antenna parameters, including all pertinent electrical, dimensional, operational, and environmental factors. It is concluded that there is a particular frequency for a specified operational depth which gives a maximum submerged-loop pickup capability for a fixed field in air and that there is a particular loop-antenna height corresponding to a given frequency which also gives a maximum submerged pickup capability for a rectangular open-core loop; however, a determination of an optimum loop-antenna design for submerged reception requires further study and experimentation.

PROBLEM STATUS

This is a final report on one phase of the problem; work on other phases is continuing.

AUTHORIZATION

NRL Problem R01-08
Project NR 414-000, Task NR 414-002
NRL Problem R01-09
BuShips No. S-1615
Project NE 021-500-3.10

Manuscript submitted September 9, 1959.

A STUDY OF VLF COMMUNICATION-SYSTEM PARAMETERS AS RELATED TO SUBMERGED-RECEPTION CAPABILITY AND RANGE PREDICTION
[Unclassified Title]

INTRODUCTION

Underwater radio communication has so far relied almost exclusively on very-low-frequency (vlf) radio waves, since radio-wave propagation in water has been found to be more effective at the longer wavelengths. When it is practical to construct higher powered stations with larger transmitting antennas (e.g., a quarter-wavelength at 30 kilocycles is about 8200 feet in air), reliable undersea reception with ranges of several thousand nautical miles may be obtained at vlf (3 to 30 kc). The phenomenon of underwater propagation of vlf radio waves has been known for many years, and the fundamental limitations encountered in subsurface radio reception have previously been investigated at NRL.[1] These limitations, very briefly, are as follows: (a) only a small fraction of the radio (electric) field existing immediately above the surface of the water appears just beneath that surface, (b) the exponential rate of rf attenuation with increasing depth is relatively large, and (c) at vlf the attenuation of the radio field in the water becomes greater as the frequency becomes higher and/or the water conductivity increases.

A theoretical estimate of vlf communication-system range based on the approximate sensitivity of the omnidirectional loop system currently in use aboard most U.S. submarines[1] should prove useful as a basis for further system improvement. The factors which need to be considered in calculating the signal voltage induced in a loop antenna submerged in water are interrelated. The influence of several design parameters, such as the loop-antenna inductance, Q, core material, etc., have been treated previously in considerable detail (1-5). However, the differences in radio-wave propagation with respect to the air and water media, and the manner in which these differences (including interface phenomena) affect the final output signal level, as well as the loop-output dependence on frequency, water salinity, relative loop-antenna dimensions, etc., have not received the detailed attention which is so essential to making reliable calculations of overall system capacity and effectiveness. Particularly needed has been the development of a mathematical expression relating all of the various major parameters affecting the reception of vlf radio signals pertinent to an estimation of system capability. Figure 1 shows a pictorial presentation of the parameters involved in shore-to-sub radio communication.

The late Dr. O. Norgorden of NRL established a basic mathematical background (6) for calculating the effects of many of the parameters involved in the reception of vlf radio signals in water for rectangular open-core loop antennas. Since his work has been largely substantiated by actual field measurements, the rectangular water-core (air-core in air) loop antenna has been chosen as the basis for the analytical model used in this report. His treatment, suitably modified and extended, has thus been used as a foundation for many of the calculations.[2]

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[1] The AT-317/BRR loop system has a 6-inch nominal diameter, 28-turn, omnidirectional, iron-core loop, which in this report is approximated by an open-core loop with a 12-inch diameter and about the same number of turns. This open-core loop serves as a basis around which a specific analytical model of a vlf communication system is formed.

[2] The majority of the data to be presented in this report were obtained from calculations made with the aid of the NAREC (NRL High-Speed Electronic Digital Computer).

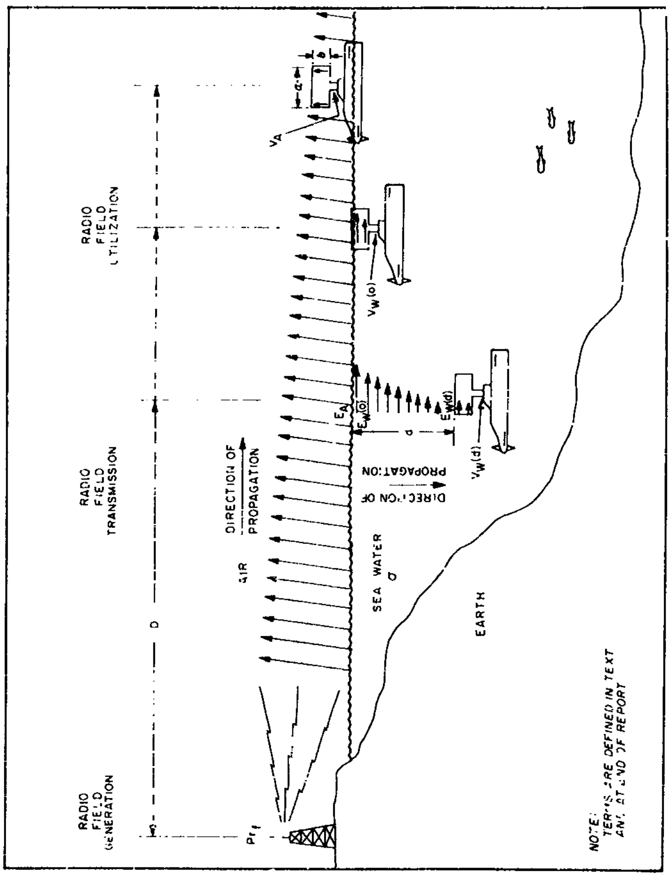

Fig. 1 - Shore-to-sub radio communication

The primary intent of this report is to relate the various communication-system parameters affecting the submerged reception of radio signals and to indicate their effect on the maximum obtainable communication range for a given depth of submergence. This report concludes the work on one phase of a continuing study of analytical techniques for the theoretical evaluation of submerged communication-system circuits and elements.

I. CHARACTERIZATION OF THE SYSTEM ELEMENTS

GENERAL SYSTEM CONSIDERATIONS

Interrelation of the Elements

The various empirical equations which have been established to predict the radio field in air at various distances from a vlf transmitter are usually expressed in terms of the transmitting-antenna down-lead current. In an overall communication-system analytical study, it is generally more meaningful to establish the vlf transmitter characteristics in terms of the radiated power rather than down-lead current. Unfortunately, vlf radiated power cannot be directly measured so easily as a parameter such as antenna input current. Therefore, as a practical matter, reliance must be placed upon certain theoretical mathematical relationships which have been developed to relate vlf antenna parameters and the empirical field equations in terms of radiated power. The premise upon which these relationships are based is that for the radiated power to remain essentially fixed over the vlf range, the transmitting-antenna down-lead current must decrease in a direct proportion to the increase in frequency (Appendix A).

The equations yield the result that for any given radiated power, the lower frequencies propagate further for a given attenuation than do the higher frequencies. For example, according to a modified form of the empirical equation originally developed by Baldwin and McDowell of the Bureau of Ships (in terms of radiated power rather than down-lead current), the field strength which is obtained at 11,500 nautical miles for a given value of radiated power from a transmitter at 10 kc will be obtained at about half that distance (5700 nautical miles) at 30 kc.

The loss in electromagnetic field strength between air and sea water (termed the field-interface loss) is very great (e.g., 1897:1 for the electric-field component at 20 kc). Fortunately, this large loss is very nearly compensated by a gain in the signal-collection capability of a loop antenna in water relative to its capability in air. For example, with the top of a one-foot-square loop just below the sea surface, the computed induced voltage in the loop at 20 kc is less than 10 percent below the induced voltage in the same loop in air immediately above the water surface. The effective voltage loss due to transition from one medium to the other is thus usually rather small, since the loop can be designed so as to minimize any change of its impedance which may occur with submergence.

The principal factor limiting the usefulness of radio waves in sea water, then, is the large attenuation of the radio field from the surface value with increase in depth. The attenuation rate for a given increase in depth becomes greater as the frequency of the radio wave increases. However, for a given amount of radiated power in the vlf range of 10 to 30 kc, the voltage induced in a small loop antenna can be expected to increase with frequency down to about 10 feet, because the decrease in field-interface loss, coupled with the increase in loop-collection capability in water with increasing frequency, outweighs the increase in loss with depth. At loop depths greater than about 10 feet, however, the induced voltage decreases as the frequency is raised, because the cumulative exponential attenuation of the radio field with depth at the higher rate which occurs with an increase in frequency reverses the rising trend of induced loop voltage which occurs at the shallower depths with an increase in frequency. Thus there might appear to be an optimum frequency for operation at any particular given depth (with a given size loop); however, all of the elements of the total system must be considered before reaching any final conclusion in this regard.

Signal-Frequency Selection

The required maximum depth of antenna submergence at a desired maximum range will be a major governing factor in the choice of signal frequency. Such elements of the system as the transmitter radiated power, the radio-wave propagation attenuation in air, the receiving-loop-antenna collection capability in air, the overall receiving-system sensitivity in air, the field-interface loss, the attenuation with increasing submergence, the receiving-antenna signal-collection capability in sea water relative to that in air, as well as other receiving-system performance characteristics with submerged operation (all diagrammed in Fig. 2), and a loss which allows for the effects of system deterioration in actual operation enter into the determination of communication-system performance. When all of the characteristics and parameters of the system are taken into consideration operational depths and/or range can be shown to increase with a decrease in frequency for a given amount of radiated power. However, transmitter construction and operating costs at vlf are closely related to operating frequency and radiated-power capability; generally, the lower the frequency and/or the greater the radiated-power capability, the higher the cost. Therefore the selection of the frequency is perhaps the most important single determination in the establishment of an effective submerged radio-communication system.

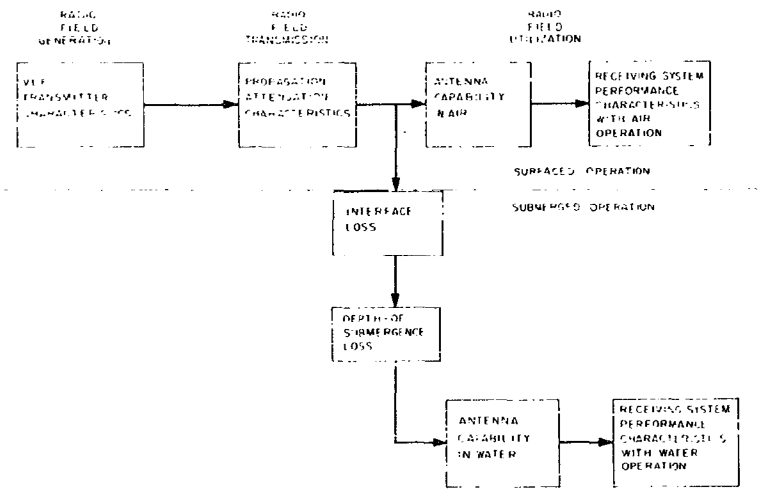

Fig. 2 - System factors associated with submerged vlf radio communication

RADIO FIELD GENERATION – VLF Transmitter Considerations

Present U.S. Navy transmitters covering the vlf portion of the radio spectrum have approximate effective radiated-power capabilities of between 20 and 300 kilowatts, depending upon the particular transmitter employed. These transmitters radiate vertically polarized waves. Many of the transmissions are cw telegraph (keyed-carrier) at keying speeds of about 20 words per minute. Increasing the effective radiated power of these transmitters will, of course, increase the range or distance from the transmitter at which the radio signals can be detected or copied. Any increase in power will generally be most effective at the lower frequencies in terms of increasing the maximum range capability, primarily because of lower propagation attenuation at lower frequency at most ranges in both air and water.

The planning for vlf transmitter installations with effective radiated-power capabilities on the order of one megawatt, plus the convenience of using this figure as a reference value in graphs and computations, have led to its use in the range calculations given in this report. However, the problems associated with the design, construction, and operation of a transmitter with such output-power capability are not considered in this report.

RADIO FIELD TRANSMISSION

VLF Propagation Attenuation in Air

Some peculiarities of radio-signal propagation in the vlf band are still not thoroughly understood, despite more than fifty years of experience by the radio profession in the use of these frequencies for communication. However, it is possible to predict the approximate field intensity as a function of distance from certain empirical equations.

Figure 3 shows the results of a computation of the empirically predicted field intensity in air, E_A, in volts per meter versus distance in nautical miles over sea water at frequencies of 10, 15, 20, 25, 30, and 100 kc, as produced by one megawatt radiated from an antenna located near a sea coast. The curves of Fig. 3 have been computed from the following equation:

$$E_A = \frac{5.10 \times 10^{-3} \sqrt{P_r}}{D} \times \epsilon^{-1.3 \times 10^{-8} f D} . \qquad (1)$$

where

P_r is the radiated power, in watts

D is the distance from the transmitting antenna, in nautical miles

ϵ is the base of natural logarithms = 2.71828. . .

f is the signal frequency, in cycles per sec

E_A is the field intensity in air, in volts per meter (v/m).

This equation is a modified form of the empirical Baldwin-McDowell equation, which is based on field data obtained on signals from vlf transmitters with known antenna effective heights and measured down-lead currents.[3] It has been found to produce computed values which, in general, agree fairly well with field-strength measurements at various distances from the transmitter for frequencies below about 100 kc.

[3] Appendix A gives a derivation of the modification.

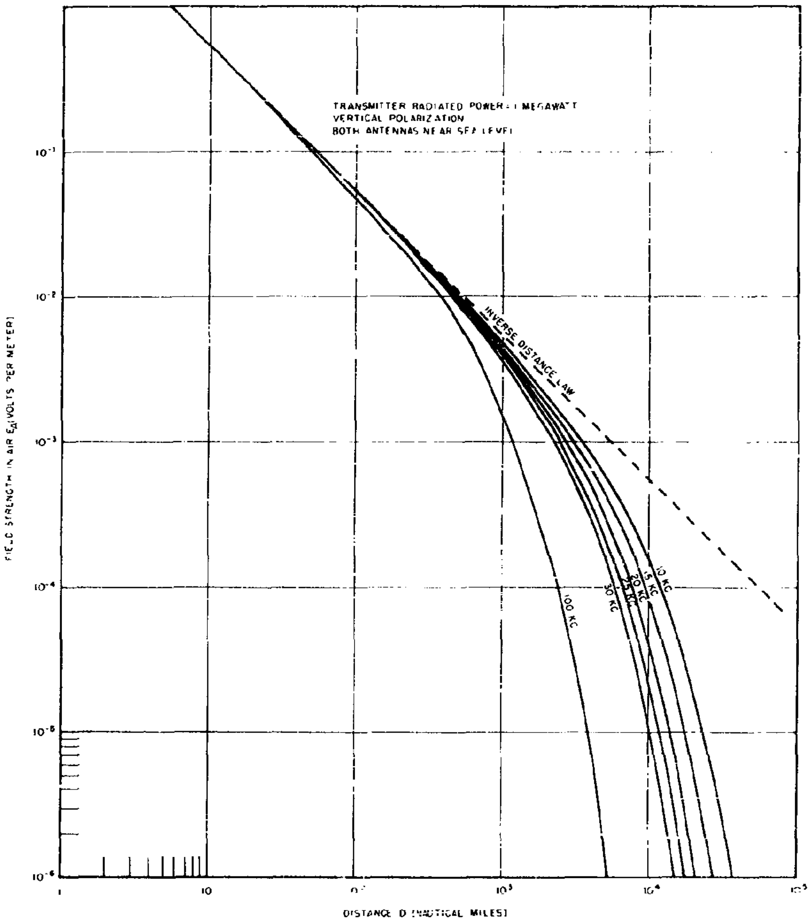

Fig. 3 - Radio field strength over sea water in the region just above the surface as a function of distance from a vlf transmitter. Computed from the empirical expression:

$$E_A = \frac{5.10 \times 10^{-3} \sqrt{P_r}}{D} \times \epsilon^{-1.3 \times 10^{-8} f D}$$

where

E_A = Field strength in air (v/m)
P_r = Transmitter radiated power (watts)
D = Distance from transmitting antenna (naut mi)
f = Signal frequency (cps)

Other empirical equations for this frequency range have been developed over the years (Austin-Cohen and Espenschied-Bailey), but the field strengths predicted by them appear to be pessimistic in comparison with scattered operational reports of radio field intensities measured at various ranges by units of the U.S. Fleet. Equation (1) is perhaps a little on the optimistic side in certain regions, but it is considered to fit the limited operational data available to NRL somewhat better than some other theoretical data (7), while it is still not as optimistic as the expression developed by Pierce (8,9). Destructive interference between the ground wave and the first-hop sky wave (e.g., at about 260 or 300 naut mi for 20 kc) usually occurs with a resulting field intensity in the interference region on the order of 10 db or more below that predicted by Fig. 3. For the purpose of this study, this loss has been lumped with other miscellaneous system losses which prevent attainment of theoretically ideal reception, although it is realized that a more detailed treatment would be useful. At distances beyond about 500 naut mi, some signal enhancement by sky wave may occur, but is probably undependable, because of variation in conditions with range and time. Pierce (9) puts the average sky-wave level roughly 6 db above the curves of Fig. 3 in the far-range region. The pessimism of Fig. 3 relative to Pierce's curves is considered to be desirable, however, because of sky-wave uncertainty.

The first factor of Eq. (1), $5.10 \times 10^{-3}\sqrt{P_r}/D$, indicates that the field intensity varies inversely with distance, as it would in free space. However, because of the earth's absorption of radio energy and the curvature of its surface, other factors, included in the $\epsilon^{-1.3 \times 10^{-8} fD}$ factor, come into play, resulting in the departure from free-space attenuation indicated by the solid-line curves of Fig. 3. For example, the 20-kc field-intensity curve follows the inverse-distance law out to a range of approximately 150 nautical miles. Beyond this point, a much more rapid decrease in field intensity occurs. The curve shows that a 20-kc transmitter radiating one megawatt in the form of vertically polarized waves over sea water should provide a field intensity of more than 100 μv/m out to about 7400 nautical miles.

Increased range may be obtained by increasing either the effective radiated power of the transmitter or the effective sensitivity of the receiver. The latter improvement will, of course, be useful only when the ambient noise external to the receiving system is, on the average, less than "internal" noise. As noted previously, any increase in either power or sensitivity will in general be more useful at the lower frequencies in terms of increasing the range of communications. This can be shown from the data of Fig. 3. For example, at the 1000-μv/m level, the range is 2,200 naut mi at 30 kc and 3,300 naut mi at 10 kc. If the field intensity is increased by a factor of ten (100:1 increase in transmitter effective power), then the 20-db power increase will simply increase the output signal-to-noise ratio of the receiving system by 20 db at both 10 and 30 kc (at the above-specified ranges). If, however, the 20-db power increase is employed to extend the range of communications, by stipulating that the 1000-μv/m field and the original receiving-system output S/N-ratio level be maintained as the threshold-sensitivity criterion, then the divergence of the 10- and 30-kc propagation attenuation curves indicates that the increase in range at the lower frequency is greater than (attenuation is less than) that at the higher frequency. In this instance the increase would be from 2200 to about 5700 naut mi at 30 kc, and from 3300 to about 11,500 naut mi at 10 kc, a relative advantage of about 2.34:1 in favor of the 10-kc signal for a 10:1 increase in field intensity.

Allowance for Miscellaneous Operational System Losses

Any estimate of overall system performance must include various operational effects which result in system losses which cannot be avoided. For example, the sky wave (as previously mentioned) can cause signal-interference effects. Another typical loss is

deterioration due to frequency misalignment or drift. For example, high atmospheric noise levels can limit range of reception by masking the desired signal, and antenna-pattern deficiencies can restrict radiation or response.

For the purposes of estimation, it is convenient to lump these various separate losses together. This resultant will be defined as the system loss which prevents attainment of ideally possible performance in the operational system. This loss l_s can be expressed as

$$l_s = \frac{E_0}{E_s}, \tag{2}$$

where

E_0 is the radio field intensity in air at a range D necessary for a reference value of receiver-output signal-to-noise ratio under ideal conditions, in volts per meter

E_s is the field intensity required under actual system operational conditions for the same output S/N, in volts per meter.

The system operational-loss allowance L_s may be expressed in decibels as

$$L_s = -20 \log_{10} l_s. \tag{3}$$

Field-Interface Loss

The ratio of the electric field strength of the retracted wave in water to that of the incident wave in air depends in a rather complicated manner upon the angle of wave incidence, the frequency of the radio wave, and the dielectric constant, permeability, and conductivity of each medium. Norgorden (6) approximated this ratio by a simple relation involving only the frequency and the conductivity of the water (on the premise that the approximation should be valid whenever the ratio of the conductivity σ, expressed in esu,[4] to the frequency f, expressed in cps, is greater than about 3600):

$$\frac{E_W(0)}{E_A} \approx \sqrt{\frac{f}{2\sigma}}. \tag{4}$$

where

$E_W(0)$ represents the radio field intensity just under the surface of the water (at zero depth), in volts per meter

E_A represents the field intensity in air at the water surface, in volts per meter.

[4]The unit for conductivity as expressed in electrostatic units is statmho-cm/square cm, which corresponds to $(1 \times 10^{-9})/9$ mho-meters/square meter in mks units.

Plots of this ratio for four values of water conductivity over the frequency range from 10 to 1000 kc are shown in Fig. 4. It is evident from Eq. (4) and Fig. 4 that the interface loss decreases as the signal frequency increases. For example, with average sea water ($\sigma = 3.6 \times 10^{10}$ esu), the ratio is 3.73×10^{-4} at 10 kc, but only 3.73×10^{-3} at 1000 kc, a decrease in loss or increase in absolute field strength of 10 to 1 for a 100-to-1 increase in frequency. It should be pointed out that the values of conductivity shown for the water-salinity states in Fig. 4 and Table 1 (and also in subsequent figures) are only approximate mean values; i.e., the value $\sigma = 3.6 \times 10^{10}$ esu given for sea water may range from less than 2.9×10^{10} to greater than 4.2×10^{10} esu.

As is apparent from Eq. (4), the field-interface loss ratio increases (the loss decreases) as the water conductivity decreases. At 20 kc, the interface loss ratio for sea water is 5.27×10^{-4}, whereas for Chesapeake Bay water it is 9.53×10^{-4}.[5] The simplified Eq. (4) is not sufficiently accurate for the low values of conductivity typical of fresh water at frequencies above about 2 kc, since the ratio of conductivity to frequency here becomes less than the 3600 limit value which Norgorden assumed in deriving Eq. (4). However, Fig. 4 does give some indication of the range of variation to be expected with conductivity decrease, with the data of doubtful accuracy resulting from the use of Eq. (4) being shown in dashed-line form. While admittedly a great simplification of the actual case, Eq. (4) has so far given results in fair agreement with the field-interface loss observed experimentally for frequencies near 18 kc.

Depth-of-Submergence Loss

The ratio of the magnitude of the radio field intensity in water at any depth to that just beneath the water surface (at zero depth) is termed the field depth-of-submergence loss. Norgorden (6) has given this ratio as

$$\frac{E_W(d)}{E_W(0)} = \epsilon^{-\frac{2\pi}{c} d \sqrt{f\sigma}}, \tag{5}$$

where

$E_W(d)$ represents the magnitude of the radio field at a depth d in feet below the water surface, in volts per meter

$E_W(0)$ represents the magnitude of the radio field at zero depth, in volts per meter

c represents the velocity of light in air, in feet per second.[6]

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[5] The field-interface loss is here expressed in terms of the ratio $E_W(0)/E_A$, rather than in decibels, because by original definition the decibel is an expression of power ratio. The expression $20 \log_{10} E_W(0)/E_A$ should not be considered as the field-interface loss expressed in decibels as originally defined, since that would imply that the impedances of the two media were the same (which, of course, is not the situation in this case). However, the right-hand ordinate scale of Fig. 4 is shown in decibels, because the introduction of normalizing factors has, in effect, canceled the impedance variation. The factors are more fully developed later in the report on page 34.

[6] Appendix B gives more accurate expressions for Eq. (5) and α which are applicable at those frequencies where the relative dielectric constant of the media must be taken into account.

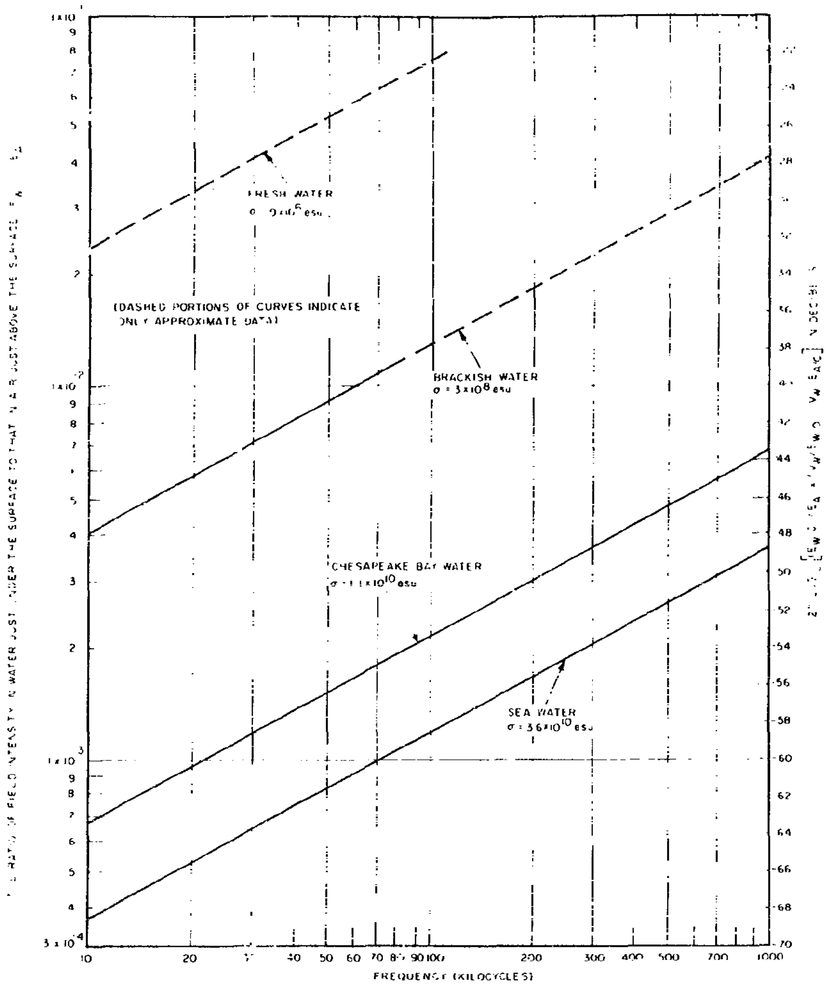

Fig. 4 - Radio electric-field interface transition loss. $E_W(0)/E_A \doteq \sqrt{f/2\sigma}$, where f is frequency in cps and σ is conductivity in esu (accurate where $\sigma/f \geq 3600$).

Table 1
Conductivities of Various Water Types

Water Type	Conductivity, σ (esu)
Sea	3.6×10^{10}
Chesapeake Bay	1.1×10^{10}
Brackish	3×10^{8}
Fresh	9×10^{8}

This loss is the principal factor limiting the propagation of radio waves in water. As indicated by Eq. (5), it is a relatively complicated function with respect to water conductivity and frequency as well as depth. Equation (5) may be transformed to a more convenient and useful expression:

$$L_d = \alpha d = -20 \log_{10}\left(\frac{E_W(d)}{E_W(0)}\right) , \tag{6}$$

where

L_d represents attenuation, in decibels

α is the rate of attenuation per foot of submergence, in db/ft.

$$\alpha \doteq 0.555 \times 10^{-7} \sqrt{f\sigma} . \tag{7}$$

Figure 5 shows α as a function of frequency over the range of 1 to 1000 kc, for the values of water conductivity previously given in Fig. 4 and Table 1. The curves indicate how the rate of attenuation increases with increasing frequency and increasing water conductivity. For example, at 20 kc, the loss in sea water is about 1.5 decibels per foot of submergence, while at 40 kc, the loss is about 2.1 decibels per foot. On the other hand, in Chesapeake Bay water, the attenuation rate is only about 0.8 decibel per foot at 20 kc.

RADIO FIELD UTILIZATION

Loop-Antenna Signal-Collection Capability in Air

The coil- or loop-type collector has so far been found to be the most useful and effective form of antenna for radio reception at considerable depths of submergence. The signal-collection capability of a loop antenna in air may be expressed by the ratio V_A/E_A, where V_A is the loop open-circuit terminal voltage (loop "induced voltage") and E_A is the radio field strength in volts per meter. For a rectangular open-core loop in air, oriented so that its plane is parallel to both the direction of wave advance and the electric

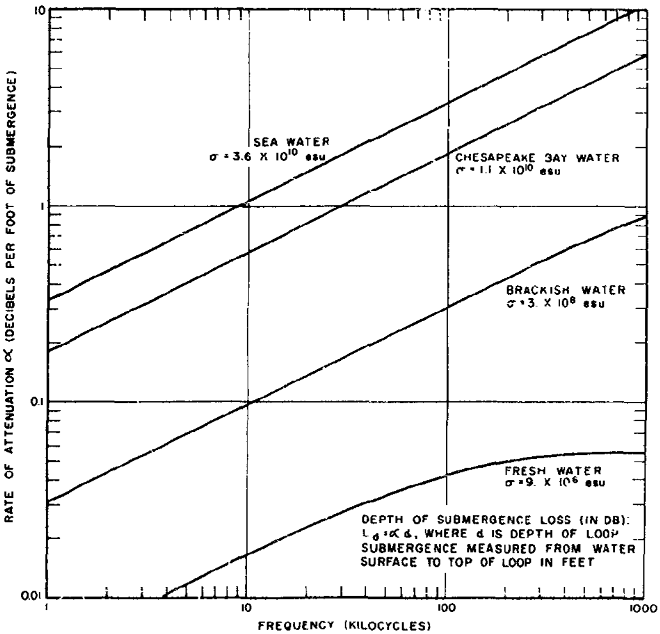

Fig. 5 - The rate of attenuation of the underwater radio field with depth as a function of frequency. α in db/ft = $0.555 \times 10^{-7}\sqrt{f\sigma} \times \sqrt{\sqrt{x^2+1}-x}$, where

$x = \kappa f/2\sigma$
$\kappa = 81$
f is in cps
σ is in esu.

vector of the oncoming wave in air (Fig. 6b),[7] the signal-collection or pickup capability (Appendix C) may be expressed as

$$\frac{V_A}{E_A} = nbk \sqrt{2\left(1 - \cos \frac{\omega a}{c}\right)}, \tag{8}$$

where

$$b << c/f \text{ or } \lambda$$

n represents the number of turns comprising the loop winding

a represents the effective loop-winding dimension parallel to the direction of wave advance, e.g., the effective loop horizontal length per turn, in feet, for a vertically polarized wave in air

b represents the loop-winding effective dimension parallel to the electric vector of the wave, e.g., the effective loop vertical height per turn, in feet, for a vertically polarized wave in air

ω represents the electrical angular velocity of the wave in radians per second ($\omega = 2\pi f$, where f is the frequency expressed in cps)

k is a conversion factor, 0.3048 meters per foot.

Since loop-antenna signal-collection capability is a function of loop dimensions as well as signal frequency, it is advantageous to standardize a particular set of dimensions to facilitate the presentation of data calculated by the use of Eq. (8). For this reason, a reference open-core loop R, consisting of a single one-foot-square turn, has been established in this report as the standard for comparison of performance capability of rectangular loops of different dimensions. Figure 6a shows the reference-loop antenna's signal-collection capability in air with respect to the factor $(V_A/E_A)_0$, which refers to a convenient reference-induced voltage of one volt and a reference field of one volt per meter in air.[8] Examination of Fig. 6a shows that for any given field strength in air, the loop-induced voltage increases in direct proportion to the increase in signal frequency for this size of loop in the frequency range shown. For example, at 10 kc, the loop-induced voltage relative to 1 volt and a 1-volt-per-meter field in air is -94.2 decibels (19.5 μv); at 20 kc, it is twice as much, or -88.2 decibels (38.9 μv), while at 40 kc it is four times as much, or -82.2 decibels (77.5 μv).

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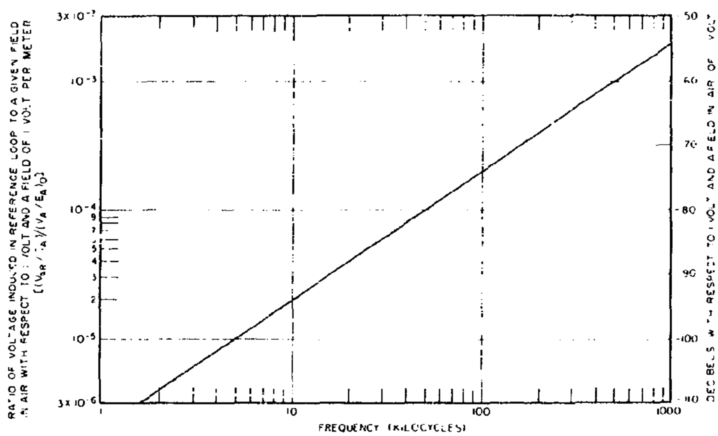
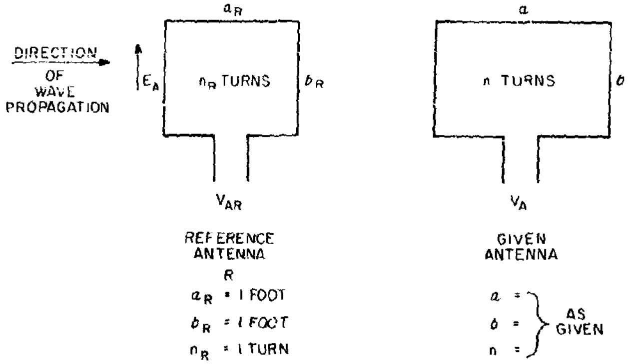
[7] Unless otherwise specified, whenever loop open-circuit terminal voltage (induced voltage) is referred to in this report, it should be understood to be the resultant of the vector sum of the potentials induced in the loop conductors when its plane is oriented in this manner. Furthermore, only vertically polarized ground waves are assumed; thus the top of the rectangular loop with length a is assumed to be always horizontal and level with the surface, and the sides vertical - including the case when the loop is submerged. The very slight error introduced by the slight forward tilt of the electric vector in the direction of propagation is neglected.

[8] A discussion on p. 34 gives further details concerning the choice of reference.



(a) Reference-loop signal-collection capability in air for reference one-foot-square, single-turn, air-core, loop R as a function of frequency (with respect to an induced voltage of one volt and a field strength in air of one volt per meter. $(V_A/E_A)_0$ refers to a convenient arbitrary reference induced voltage of 1 volt and a field of 1 volt per meter.

Fig. 6 - Loop-antenna collection capability in air - induced (or open-circuit terminal) voltage produced by a given field strength in air



(b) Extension of the treatment of loop collection capability in air to include loops with dimensions and number of turns different from those for the reference loop.

Fig. 6 (Continued) - Loop-antenna collection capability in air - induced (or open-circuit (terminal) voltage produced by a given field strength in air

NOTE: The collection capability of any given loop antenna in air may be expressed in terms of the reference one-foot-square, single-turn, air-core loop by defining a factor F_{CA} as follows:

$$F_{CA} \times \left(\frac{V_A}{E_A}\right)_R = \left(\frac{V_A}{E_A}\right), \quad F_{CA} = \frac{V_A}{V_{AR}} = \frac{nb}{n_R b_R}\sqrt{\frac{1-\cos\frac{\omega a}{c}}{1-\cos\frac{\omega a_R}{c}}}$$

or expressed in decibels,

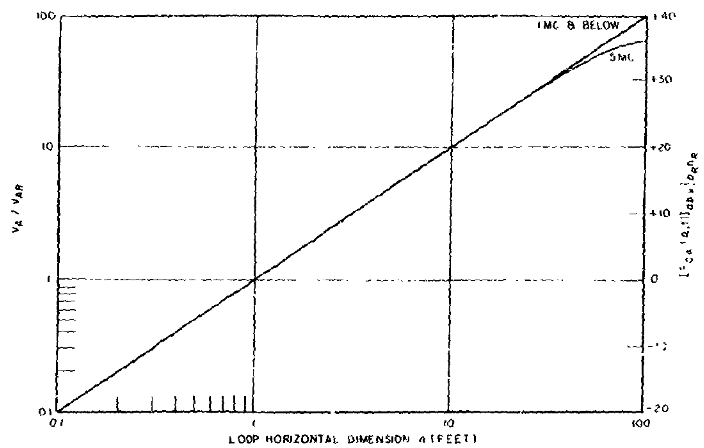
$$\left[F_{CA}(a, b, n, f)\right]_{dbv} = 20 \log_{10} \frac{V_A}{V_{AR}}.$$

In order to simplify the presentation, it is convenient to plot a curve showing only the effects of variation of f and a by normalizing with respect to n/n_R and b/b_R; thus

$$\left[F_{CA}(a, b, n, f)\right]_{dbv} = \left[F_{CA}(a, f)\right]_{dbv}\Big|_{b_R n_R} + 20 \log_{10}\left(\frac{n}{n_R}\right)$$

$$+ 20 \log_{10}\left(\frac{b}{b_R}\right)$$

where $\left[F_{CA}(a, f)\right]_{dbv}\Big|_{b_R n_R}$ is shown in Fig. 6c.



(c) Relative loop-antenna signal-collection capability in sea water for several frequencies as a function of the loop horizontal dimension

Fig. 6 (Continued) - Loop-antenna collection capability in air - induced (or open-circuit terminal) voltage produced by a given field strength in air

NOTE: Inductance and Q have not been required to remain fixed with change of a, b, and n relative to reference loop R.

The signal-collection capability of any given loop antenna in air may be expressed relative to that of the reference loop R by a factor F_{CA}, as follows:

$$F_{CA} = \left(\frac{V_A}{E_A}\right)_R \div \left(\frac{V_A}{E_A}\right), \text{ hence } F_{CA} = \frac{V_A}{V_{AR}} , \tag{9}$$

where V_A and V_{AR} are the induced voltages generated in the two loops when in any (same) given field in air. This ratio should, of course, hold for any other field intensity, except perhaps where secondary effects, such as iron-core saturation, occur. Equation (8) shows that the loop-antenna collection capability in air is directly proportional to dimension b and the number of turns n and also is a more complex function of the frequency f and the dimension a. However, for frequencies less than about 1 Mc and dimensions of a less than about 100 feet, the loop pickup capability in air is for practical purposes directly proportional to a, b, n, and f; i.e., a ten-times increase in either the a or b dimension, the number of turns, or the frequency will result in a 20-decibel (dbv)[9] increase in the loop-antenna collection capability in air. When $fa \ll 5.4 \times 10^8$ and $b \ll \lambda/4$, Eq. (8) can be very closely approximated by the simpler relation

$$\frac{V_A}{E_A} = \frac{nbk\omega a}{c} . \tag{10}$$

The factor F_{CA} in Eq. (9) may be expressed in decibels (dbv) as follows:

$$\left[F_{CA}\ (a,\ b,\ n,\ f)\right]_{dbv} = 20\ \log_{10}\ (V_A/V_{AR}) . \tag{11}$$

In order to simplify the presentation of this factor, it is convenient to plot a curve showing only the effects of variations of f and a by normalizing with respect to n/n_R and b/b_R where n_R and b_R refer to the reference loop; thus

$$\left[F_{CA}\ (a,\ b,\ n,\ f)\right]_{dbv} = \left[F_{CA}\ (a,\ f)\right]_{dbv}\Big|_{b_R n_R} + 20\ \log_{10}\left(\frac{b}{b_R}\right) + 20\ \log_{10}\left(\frac{n}{n_R}\right). \tag{12}$$

Figure 6c shows how the term $[F_{CA}\ (a, f)]_{dbv}\big|_{b_R n_R}$ changes for various a dimensions.

[9] This notation has been adopted throughout this report to differentiate between the true decibel, which is an expression of power ratio, and a restricted "decibel" which refers to a voltage ratio, with the impedance aspects being treated as a separate loss or gain term: i.e.,

$$\begin{aligned} db &= 10\ \log_{10}\ (P_1/P_2) \\ &= 10\ \log_{10}\ (V_1^2/r_1)/(V_2^2/r_2) \\ &= 20\ \log_{10}\ (V_1/V_2) - 10\ \log_{10}\ (r_1/r_2) \\ &= \underline{dbv} - dbr \end{aligned}$$

Ferrous-core materials are often used to improve the collection capability of loop antennas. Experimental studies (5) have shown that for in-air operation and a given size of loop winding, about 6 decibels increase in signal output can be realized for a given value of loop inductance with iron cores not much greater in volume than the enclosed air space of the winding (see footnote 1).

Receiving-System Performance Characteristics for In-Air Operation

The sensitivity of a radio receiving system employing a loop-type antenna is usually specified in terms of the field strength in air necessary to provide a standard level of output and output signal-to-noise ratio from the receiver. The design of any particular receiver will be governed by the modulation type, rate of transmission, and bandwidth of the signals to be received, the receiver output power levels required, and other factors. Present Navy vlf signals are mainly of the keyed-carrier (cw telegraph) type transmitted at keying speeds seldom greater than about 20 words per minute. Receiver output bandwidth is about 200 cps, and the standard output power level specified for performance measurement is usually 6 milliwatts (developed in a 600-ohm or other specified output load) with 20 decibels output signal-to-noise ratio $(S/N)_o$.

Design threshold sensitivity S_0, as referred to in this report, applies particularly to Morse code telegraph and is that cw input signal voltage which, when heterodyned by the audio-beat oscillator of a tone-telegraph receiver, will produce zero-decibel signal-to-noise ratio at the receiver output load at the standard output power level. The 20-decibel $(S/N)_o$ usually specified by the Navy for standard cw sensitivity measurements, S_{20}, is excessively good for end-of-range estimation in a keyed cw system, since a capable operator can easily read the noisier S_0 signal. The loop-induced voltage V_0 (generated by a field E_{A_0}) which produces zero-decibel $(S/N)_o$ in the output of the associated receiver is referred to as the design-threshold "voltage-sensitivity" figure for the entire receiving system, and the field E_{A_0} is defined as the design-threshold "field sensitivity."

The principal reason for specifying receiving-system performance capability with the loop antenna in air is that performance measurements are much easier to make (and therefore are much more rapidly made) with the loop in air than are measurements with the loop antenna submerged. Additionally, such in-air measurements show the receiving system's performance capability with surface craft (e.g., a surfaced submarine).

Sensitivity measurements made in terms of field strength necessarily include the loop antenna's contribution to receiving-system performance as a collector, whereas voltage-sensitivity measurements (usually made with a signal generator and dummy-loop circuit simulating the impedance of an actual loop) do not include loop signal-collection capability as a factor. The specified performance, of course, must be achieved by the combination (referred to as the "receiving system") of the receiver proper, the loop-to-receiver interconnecting cable (including any coupling devices), and the loop antenna proper. Field-sensitivity measurements are feasible for indicating the performance capability of receiving systems employing loop antennas operated in air, because here the antenna parameters are under control of the receiver designer. Consequently, the sensitivity is often stated in terms of the field in air necessary to provide a particular level of output signal and output signal-to-noise ratio with the loop in air. However, it will be shown subsequently that although such a practice may be acceptable for surfaced (in-air) operation, it does not necessarily provide an indication of the field necessary to obtain equivalent performance with submerged operation.

Loop-Antenna Signal-Collection Capability in Water

The signal-collection capability of a loop antenna totally immersed in water may be expressed as the ratio of its induced, or open-circuit, terminal voltage V_w to the underwater field E_w at the top of the loop, i.e., V_w/E_w. Norgorden (6) derived an expression for this ratio for a rectangular water-core loop, oriented so that its plane is parallel to the direction of propagation and normal to the surface (refer to footnote 7). Appendix D shows the derivation of a converted form of this expression, which has been used as a basis for certain calculations in this report:[10]

$$\frac{V_w}{E_w} = n\alpha k \sqrt{1 - 2\epsilon^{\theta}\cos\theta + \epsilon^{2\theta}} \; , \tag{13}$$

where

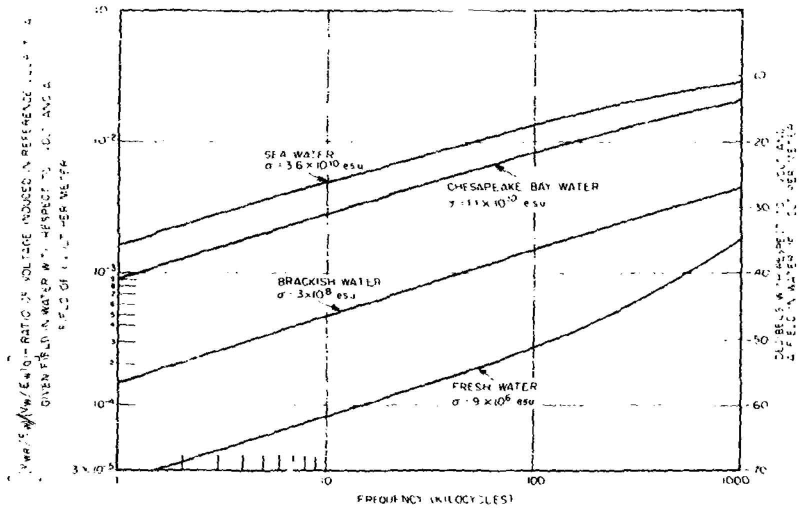
$$\theta = -\frac{2\pi}{c} b \sqrt{f\sigma}, \text{ when } \sigma/f > 3600.$$

It is apparent that (as for the case of the loop in air) the loop's signal-collection capability in water is a function of loop dimensions as well as the signal frequency. Here, it is again advantageous to standardize a particular loop to facilitate the presentation of data calculated by the use of Eq. (13). Therefore the reference one-foot-square loop R is again employed as the standard for comparing rectangular loops of different dimensions. Figure 7a shows the reference loop's signal-collection capability in water with respect to the factor $(V_w/E_w)_0$, which refers to a convenient reference-induced voltage of one volt and a reference field strength of one volt per meter in water.[11] The collection capability is plotted for the same values of water conductivity as previously given in Table 1.

It is apparent from Fig. 7a that for a given field strength in water at the top of the loop, the loop's signal-collection capability increases with both increase of frequency and increase of conductivity. This effect is caused by the increase in rate of field attenuation per unit change of depth (db per foot), which results from greater conductivity and higher frequency. The greater rate of attenuation has the effect of making the field at the bottom of the loop relatively less intense. Normally, for small (compared to 1/4 wavelength in water) vertical separation of the upper and lower elements of a rectangular loop, the voltage induced in the lower element by a wave propagating vertically downward adds in nearly opposite phase to the voltage induced in the upper element. Therefore, if the voltage induced in the lower element becomes less due to field attenuation across the span of the loop while the voltage of the upper element remains unchanged for a given level of field strength at the top of the loop, an increase in output voltage from (or effective induced voltage in) the loop will be realized. The length of radio waves in water also becomes less with increase in water conductivity because of an associated decrease in the velocity of propagation. Therefore a greater phase difference can then exist between the voltages induced in the upper and lower elements of loops with relatively small vertical dimensions, so that their phasor sum tends to increase with increase in conductivity and frequency from this cause also.

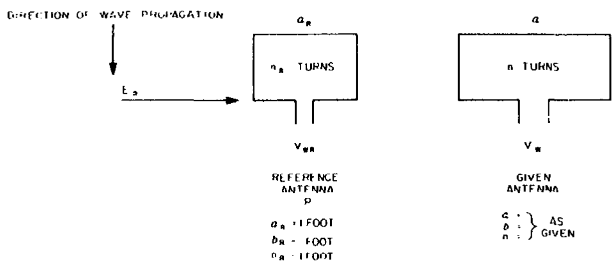
[10] The derivation of a more accurate expression which is applicable at those frequencies where the relative dielectric constant of the medium must be taken into account is outlined in Appendixes B and D.

[11] Further discussion on p. 34 gives more information regarding the choice of reference.



(a) Reference-loop collection capability in water for several salinity conditions for reference one-foot-square, single-turn, loop R as a function of frequency (with respect to an induced voltage of one volt and a field strength in water of one volt per meter). $(V_w/E_w)_0$ refers to a convenient arbitrary reference induced voltage of 1 volt and a field of 1 volt per meter.

Fig. 7 - Loop-antenna collection capability in water - induced (or open-circuit terminal) voltage produced by a given field strength in water



(b) Extension of the treatment of loop collection capability in water to include loops with dimensions and number of turns different from those for the reference loop

Fig. 7 (Continued) - Loop-antenna collection capability in water - induced (or open-circuit terminal) voltage produced by a given field strength in water

NOTE: The collection capability of any given loop antenna in water may be expressed in terms of the reference one-foot-square, single-turn loop by defining a factor F_{CW} as follows:

$$F_{CW} \times \left(\frac{V_W}{E_W}\right)_R = \left(\frac{V_W}{E_W}\right), \quad F_{CW} = \frac{V_W}{V_{WR}} = \frac{na}{n_R a_R}\sqrt{\frac{1-2\epsilon^{\theta}\cos\theta+\epsilon^{2\theta}}{1-2\epsilon^{\theta_R}\cos\theta_R+\epsilon^{2\theta_R}}} .$$

where

$$\theta = -\frac{2\pi}{c}\, b\sqrt{f\sigma}, \quad \theta_R = -\frac{2\pi}{c}\, b_R\sqrt{f\sigma}.$$

and $\sigma/f \gg 0$.

Expressed in decibels

$$F_{CW}(a,b,n,f,\sigma)_{dbv} = 20\log_{10}(V_W/V_{WR}).$$

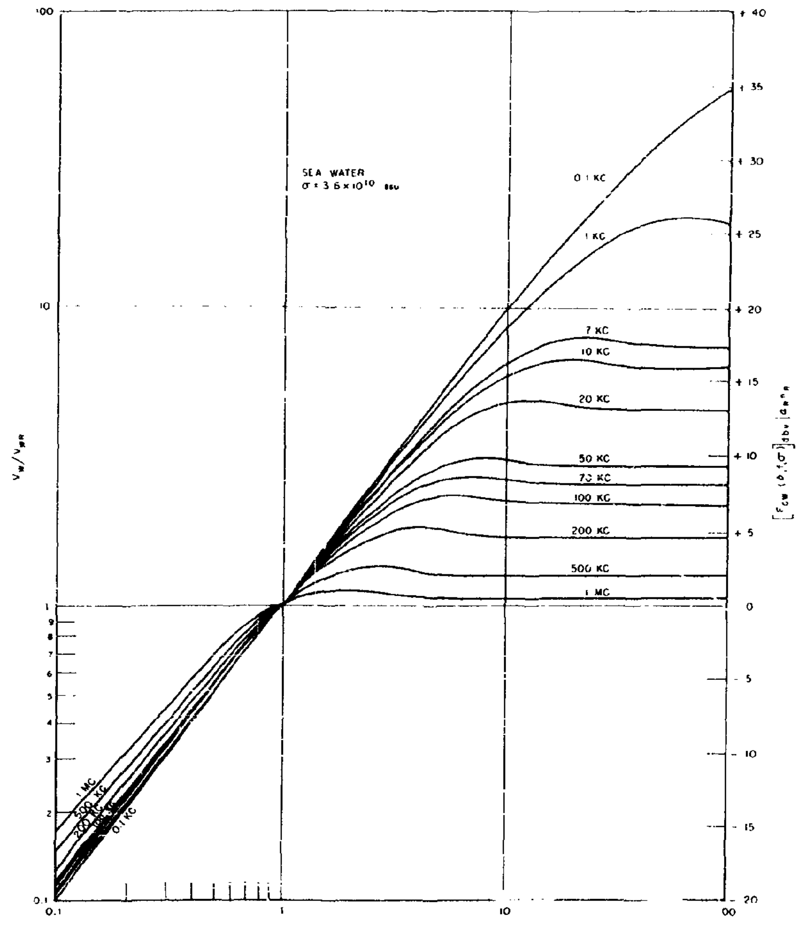
In order to simplify the presentation, it is convenient to plot a curve showing only the effects of variation of f and b by normalizing with respect to n/n_R and a/a_R; thus

$$\left[F_{CW}(a,b,n,f,\sigma)\right]_{dbv} = \left[F_{CW}(b,f,\sigma)\right]_{dbv}\Big|_{a_R n_R} + 20\log_{10}\left(\frac{n}{n_R}\right) + 20\log_{10}\left(\frac{a}{a_R}\right),$$

where

$$\left[F_{CW}(b,f,\sigma)\right]_{dbv}\Big|_{a_R n_R}$$

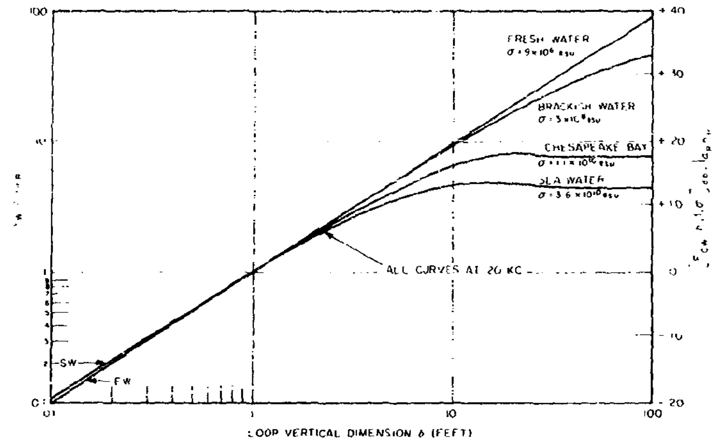
is shown in Fig. 7c.



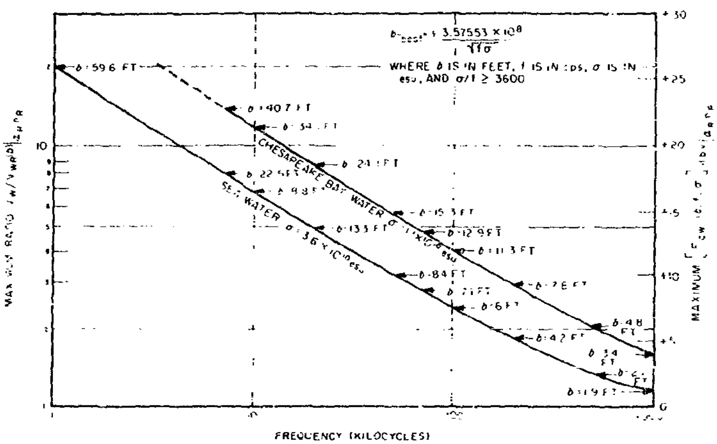
(c) Relative loop-antenna signal-collection capability in sea water for several frequencies as a function of the loop vertical dimension

Fig. 7 (Continued) - Loop antenna collection capability in water - induced (or open-circuit terminal) voltage produced by a given field strength in water

NOTE: Inductance and Q have not been required to remain fixed with change of a, b, and n relative to reference loop R



(d) Relative loop-antenna signal-collection capability at 20 kc for several salinity conditions as a function of the loop vertical dimension



(e) Maximum relative signal-collection capability achievable by employing rectangular loop antennas with "best" vertical dimensions as a function of frequency

Fig. 7 (Continued) - Loop-antenna collection capability in water - induced (or open-circuit terminal) voltage produced by a given field strength in water

It should be noted, of course, that the same increasing gradient of field intensity which causes loop pickup capability to increase in an increasingly lossy medium will cause the voltage induced in the upper element of the loop to decrease with increasing frequency and conductivity, as soon as certain depths of submergence are reached. Further submergence will then result in a rapid decrease in loop output, as will be evident subsequently (Fig. 9).

The signal-collection capability of any given loop antenna in water may be expressed relative to that of the reference loop R by a factor F_{CW} as follows:

$$F_{CW} \times \left(\frac{V_W}{E_W}\right)_R = \left(\frac{V_W}{E_W}\right), \text{ hence } F_{CW} = \frac{V_W}{V_{WR}} . \tag{14}$$

Equation (13) has shown that rectangular loop signal-collection capability in water is a function of the dimensions a and b and the number of turns n, as well as the water conductivity σ and the frequency f. The factor F_{CW} in Eq. (14) may be expressed in decibels (dbv) as follows:

$$\left[F_{CW}\ (a,\ b,\ n,\ f,\ \sigma)\right]_{dbv} = 20\ \log_{10}\left(\frac{V_W}{V_{WR}}\right). \tag{15}$$

The loop-output capability in water is directly proportional to the loop dimension a and the number of turns n, but it is a more complex function of the dimension b, the frequency f, and the conductivity σ. The presentation of the F_{CW} factor can therefore be simplified by normalizing with respect to n/n_R and a/a_R ; thus

$$\left[F_{CW}\ (a,\ b,\ n,\ f,\ \sigma)\right]_{dbv} = \left[F_{CW}\ (b,\ f,\ \sigma)\right]_{dbv}\Big|_{a_R n_R} + 20\ \log_{10}\left(\frac{a}{a_R}\right) + 20\ \log_{10}\left(\frac{n}{n_R}\right). \tag{16}$$

Curves showing only the effects of variations of b, f, and σ may then be plotted as in Fig. 7c, which shows how the term $[F_{CW}\ (b, f, \sigma)]_{dbv}|_{a_R n_R}$ changes for various b dimensions for the case of sea water. It is seen that an increase of the height or vertical side dimension b above the reference height (1 foot) produces no substantial increase in output when the frequency exceeds 1 Mc. Similarly, output at 10 kc does not increase substantially for values of b greater than about 15 feet. It will be noted that a considerable increase in collection capability can be realized with the use of loop-antenna vertical dimensions greater than one foot for frequencies below 1 Mc. The threshold effect with respect to the b dimension appears to be a function of $f^{-1/2}$. Figure 7d indicates the variation in relative loop capability to be expected with water of different salinity at 20 kc. It is apparent that the threshold effect with respect to the b dimension is a function of $\sigma^{-1/2}$. Within the range of the b dimensions for which data are shown, the actual (or absolute) antenna collection capability in water is always higher with greater conductivity, even though the relative data shown in Fig. 7d with respect to the b dimension indicates that relative output decreases as water conductivity increases. This may be seen by adding

appropriate numbers obtained from Figs. 7a and 7d to obtain the actual antenna collection capability in water corresponding to the different size loops. For example, the antenna collection capability for a loop with a 10-foot b dimension and operation in sea water at 20 kc is about -9.9 decibels (+13.5 db from Fig. 7d added to -23.4 db from Fig. 7a = -9.9 db), whereas for fresh water it is about -38.9 decibels; thus the loop collection capability in sea water is about 29 decibels higher than that for fresh water for this b dimension and this frequency. However, the sea-water advantage over water of lower conductivity does tend to decrease for the larger b dimensions. Data at other frequencies from which curves similar to the 20-kc data of Fig. 7d can be plotted are shown in Table 2.

It is evident from Figs. 7c and 7d that for each frequency and conductivity condition there is a particular b dimension which provides a maximum loop signal-collection capability. This maximum is shown in Fig. 7e, plotted as a function of the frequency for two conductivity conditions. This phenomenon is apparently caused by an antenna self-resonance effect which occurs when the loop vertical dimension is near a half-wavelength as measured in water, for which case the voltage induced in the bottom part of the loop is nearly in phase with that of the top part. According to the theoretical derivation in Appendix E, which includes both the phase and attenuation effects, the maximum occurs with the b dimension equal to 0.3635 of a wavelength, as measured in the water.

Upon cursory examination, increasing the dimension a or the number of loop turns n would appear to provide unlimited possibilities with regard to improving loop collection ability (Eq. 13). This, of course, is true within the limits imposed upon the dimension a, i.e., that it should be short compared with a wavelength in air. However, a large antenna collection capability in itself does not insure having a high energy-transfer efficiency to the receiver. The amount of power which can be extracted from the loop antenna is very much a function of the loop antenna impedance, which in turn is not directly determined by the collection capability. Furthermore, the Q and inductance of a tuned-loop antenna also vitally affect the performance capability of the receiving system. The limits to which a variation in either a, b, or n can be taken without compromising the loop electrical design and/or practical physical considerations in the water environment must be precisely charted, for these latter factors are an intimate part of the overall design picture and cannot be ignored in any realistic appraisal of potential loop-antenna usefulness.[12] The gains indicated in Fig. 7c and Eq. (13) as being achievable with the larger loop dimensions can be applied, of course, in the analysis of those communication systems that incorporate loop-antenna systems (with such correspondingly larger loop dimensions) which do satisfactorily realize the conditions imposed by practical electrical, physical, and operational criteria.

Receiving-System Performance Characteristics for In-Water Operation

With receiving-system sensitivity specified in terms of the minimum satisfactory field strength in air which provides minimum acceptable output performance, it is incumbent upon the system designer to ascertain that the performance obtained from the system will never be poorer than that specified for the receiving-system sensitivity figure. The voltage V_0 induced in the loop antenna in a field E_{A_0} which produces an in-air threshold

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[12] This charting remains as part of an overall system optimization study.

Table 2
Water Salinity Effects on F_{cw} at Various Frequencies (Loop-Antenna Collection Capability in Water for Reference Antenna is Shown in Fig. 7a, 20-kc Data is Shown in Fig. 7d)

f (kc)	σ (esu)	F_{cw} in db								
		b = 0.1 Ft	b = 0.5 Ft	b = 1 Ft	b = 2 Ft	b = 5 Ft	b = 10 Ft	b = 20 Ft	b = 50 Ft	b = 100 Ft
1	σ'	-19.850	-5.937	0	+5.854	+13.314	+18.502	+22.862	+25.983	+25.630
1	σ''	-19.917	-5.975	0	+5.929	+13.611	+19.172	+24.273	+29.486	+31.128
1	σ'''	-19.986	-6.013	0	+6.005	+13.919	+19.863	+25.732	+33.235	+38.496
1	σ''''	-19.998	-6.019	0	+6.018	+13.969	+19.976	+25.971	+33.851	+39.740
10	σ'	-19.526	-5.757	0	+5.494	+11.875	+15.288	+16.419	+15.825	+15.845
10	σ''	-19.738	-5.875	0	+5.730	+12.816	+17.384	+20.531	+21.053	+20.750
10	σ'''	-19.957	-5.997	0	+5.973	+13.787	+19.568	+25.108	+31.627	+35.263
10	σ''''	-19.993	-6.016	0	+6.012	+13.947	+19.927	+25.866	+33.580	+39.193
50	σ'	-18.941	-5.432	0	+4.844	+9.311	+10.019	+9.481	+9.506	+9.506
50	σ''	-19.414	-5.695	0	+5.370	+11.381	+14.206	+14.550	+14.129	+14.131
50	σ'''	-19.903	-5.967	0	+5.913	+13.551	+19.036	+23.986	+28.753	+29.804
50	σ''''	-19.985	-6.012	0	+6.004	+13.913	+19.850	+25.703	+33.156	+38.328
100	σ'	19.502	-5.188	0	+4.360	+7.481	+7.128	+6.982	+6.983	+6.983
100	σ''	19.172	-5.[illegible]61	0	+5.101	+10.215	+11.959	+11.449	+11.391	+11.391
100	σ'''	-19.863	-5.[illegible]45	0	+5.869	+13.375	+18.641	+23.151	+26.668	+26.511
100	σ''''	-19.981	-6.010	0	+5.999	+13.894	+19.808	+25.614	+32.912	+37.786

NOTE: $\sigma' = 3.6 \times 10^{10}$ esu ≈ Sea water
$\sigma'' = 1.1 \times 10^{10}$ esu ≈ Chesapeake Bay water
$\sigma''' = 3.0 \times 10^{8}$ esu ≈ Brackish water
$\sigma'''' = 9.0 \times 10^{6}$ esu ≈ Fresh water

sensitivity S_0 is the lowest acceptable level for satisfactory system performance and must at least be achieved under all possible conditions of range and/or antenna submergence if a satisfactory communication system is to be established.

For the purposes of this report, it is assumed that for a specified receiving system sensitivity figure, equivalent performance is obtained whenever the same voltage is induced in a loop antenna, independent of the medium (e.g., air or water) in which the antenna may be placed. It is also assumed that the equivalent source impedance remains unchanged upon submergence.[13] Presumably, this should be the normal case, since considerable care is usually taken in the design of loop systems to insure that the impedance and Q of the antenna do not appreciably change upon submergence. Thus the transfer efficiency of the loop system would not change appreciably either, and the amount of induced voltage required to obtain the specified performance should remain about the same. If the field in water at the loop antenna is sufficient to induce the same value of voltage V_0 in the loop in water as obtained when the loop is in air, then the minimum acceptable system performance criterion is presumed to have been met (and a submerged communication system providing design threshold sensitivity output is considered established).

[13]It should be mentioned that there are many other types of antenna systems for which the assumptions made above would not be true due to impedance and Q change upon submergence, e.g., the electric or probe-type antenna. This study has been largely restricted to loop-type antenna systems.

II. INTEGRATION OF THE SYSTEM ELEMENTS

IN-AIR OPERATION

System Equations for Surfaced Operation

Certainly, for satisfactory overall system performance, the field actually produced at a particular range D must be equal to or greater than the field E_s necessary for producing minimum satisfactory receiver output. This forms a basis for combining the expressions for the various elements of a vlf communication system into one unified statement, or "system equation," which contains all of the various parameters that determine and can affect overall system-performance capability. Since Eq. (1) gives a direct expression for the field strength in air which will exist at any particular range D for a given frequency and radiated power, and since E_0 in Eq. (2) is equal to E_{A_0} in this case, a system equation for surfaced operation can be established in terms of E_{A_0}, the threshold field sensitivity (measured or specified at the signal frequency) of the receiving system; accordingly,

$$E_A \geq E_s = \frac{E_0}{l_s} = \frac{E_{A_0}}{l_s}. \tag{17}$$

Applying this statement to the analytical rectangular open-core loop model which has already been developed in this report, and using the modified Baldwin-McDowell empirical formula for the field strength, gives

$$\frac{5.10 \times 10^{-3}\sqrt{P_r}}{D} \times \epsilon^{-1.3\times 10^{-8} fD} \geq \frac{E_{A_0}}{l_s}. \tag{18}$$

This same relationship may also be expressed conveniently in terms of the design-threshold voltage sensitivity of the receiving system; thus

$$E_A \geq \frac{V_0}{l_s(V_A/E_A)}. \tag{19}$$

since, by definition,

$$\frac{V_0}{E_{A_0}} = \left(\frac{V_A}{E_A}\right). \tag{20}$$

Applying the analytical model [1] to Eq. (19) gives

$$\frac{5.10 \times 10^{-3}\sqrt{P_r}}{D} \times \epsilon^{-1.3\times 10^{-8} fD} \geq \frac{V_0}{l_s\, n k b \sqrt{2\left(1 - \cos \frac{\omega a}{c}\right)}}. \tag{21}$$

IN-WATER OPERATION

Induced Loop Voltage for a Given Field Strength in Air

Before establishing a similar system equation for in-water operation, it is first necessary to determine the field E_0 in air which will induce a voltage V_0 in a submerged loop antenna corresponding to a design threshold sensitivity S_0 (consistent with the assumption previously discussed that equivalent performance is obtained whenever the same voltage is induced in the loop antenna, irrespective of the medium in which the antenna may be located). The ratio of the induced voltage $V_W(d)$ in a submerged-loop antenna in water at a specified depth of submergence d produced by a given radio field strength in air at the surface E_A to that field is then by definition equal to the ratio of V_0 to E_0; or

$$\frac{V_W(d)}{E_A} = \frac{V_0}{E_0}. \tag{22}$$

An identity may now be established which relates this ratio to several of the system elements already discussed:

$$\frac{V_W(d)}{E_A} = \overbrace{\frac{E_W(0)}{E_A}}^{\text{(Field-interface loss)}} \times \overbrace{\frac{E_W(d)}{E_W(0)}}^{\text{(Depth-of-submergence loss)}} \times \overbrace{\frac{V_W(d)}{E_W(d)}}^{\text{(Antenna collection capability in water)}}. \tag{23}$$

Upon being applied to the case of water-core rectangular loops and expanded, this becomes

$$\frac{V_W(d)}{E_A} = \sqrt{\frac{f}{2\cdot}} \times \epsilon^{-\frac{d}{b}} \times n a k \sqrt{1 - 2\epsilon \cos + \epsilon^{2}}. \tag{24}$$

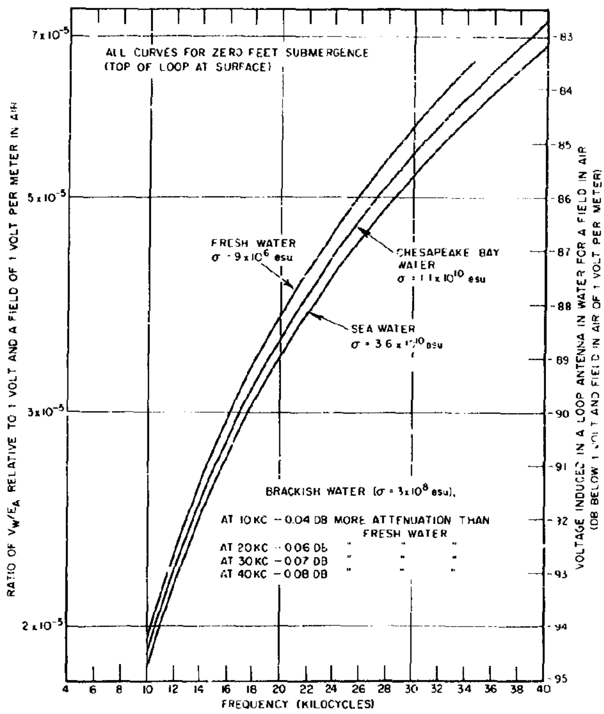
Substituting Eq. (14) in Eq. (23) gives

$$\frac{V_W(d)}{E_A} = \frac{E_W(0)}{E_A} \times \frac{E_W(d)}{E_W(0)} \times \left(\frac{V_W(d)_R}{E_W(d)} \times F_{CW}\right), \tag{25}$$

or

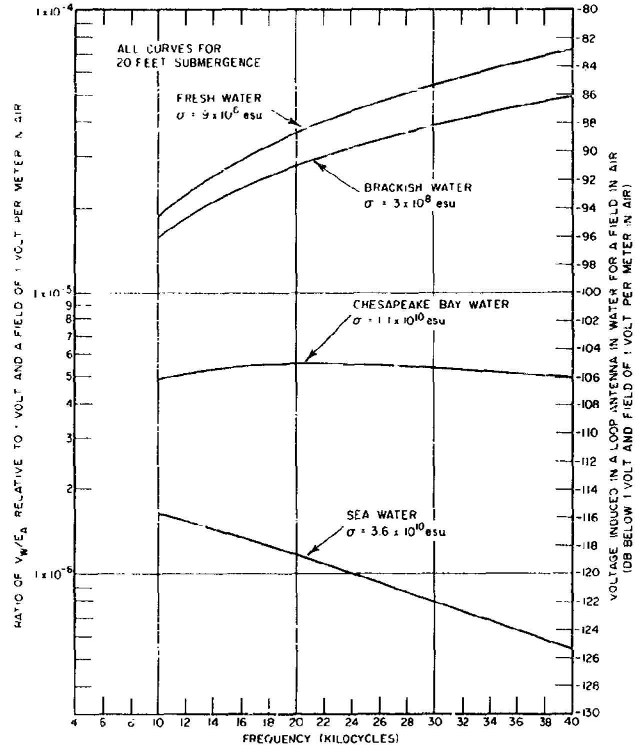
$$\frac{V_W(d)}{E_A} = \frac{V_W(d)_R}{E_A} \times F_{CW}, \tag{26}$$

since the field-interface loss and depth-of-submergence loss are both independent of antenna dimensions. Therefore the same factor, F_{CW} (as already plotted in parts C, D, and E of Fig. 7), also indicates the effect of different loop dimensions and number of turns on $V_W(d)/E_A$. Figures 8 and 9 show $(V_W(d)_R/E_A)/(V_W/E_A)_0$, the voltage induced in a reference-loop antenna in water for a given field in air expressed in decibels below 1 volt and a field in air of 1 volt per meter. The factor $(V_W/E_A)_0$ refers to the convenient reference induced voltage of 1 volt and the reference field of 1 volt per meter in air.



(a) Voltage induced in reference loop antenna when just submerged for several salinity conditions with a given field strength in air as a function of frequency (with respect to an induced voltage of one volt and a field <u>in air</u> of one volt per meter)

Fig. 8 - Effect of water conductivity on the voltage induced in reference one-foot-square, single-turn loop antenna in water at two particular depths for a given field strength in air as a function of frequency



(b) Voltage induced in reference loop antenna at 20-ft submergence for several salinity conditions with a given field strength in air as a function of frequency (with respect to an induced voltage of one volt and a field in air of one volt per meter)

Fig. 8 (Continued) - Effect of water conductivity on the voltage induced in reference one-foot-square, single-turn loop antenna in water at two particular depths for a given field strength in air as a function of frequency

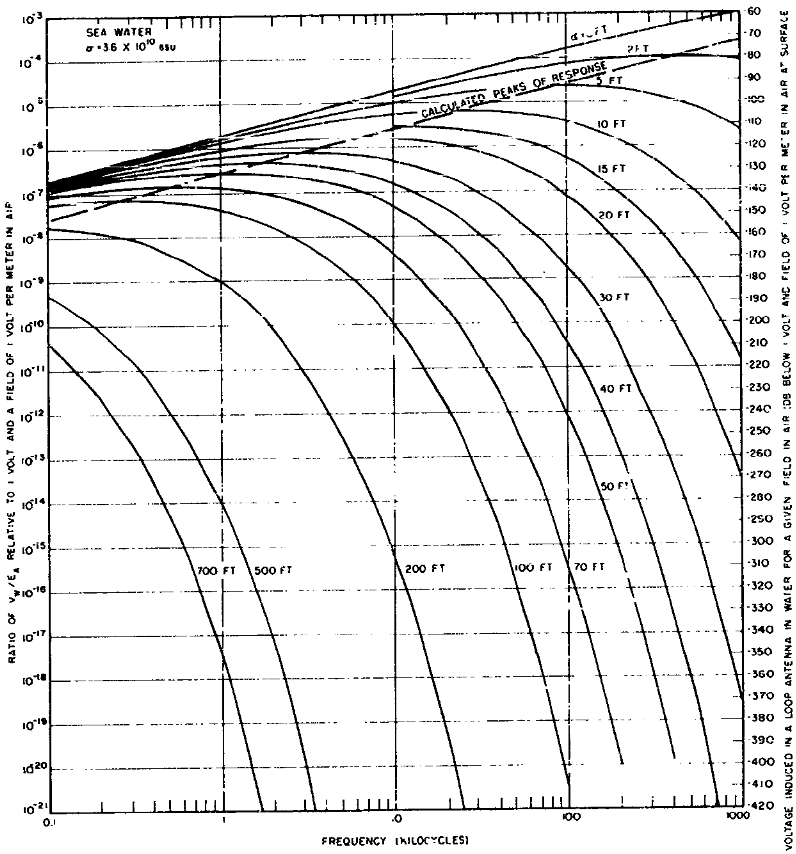


Fig. 9 - Voltage induced in reference loop antenna submerged in sea water for a given field strength in air at the surface as a function of depth and frequency (with respect to an induced voltage of one volt and a field strength in air of one volt per meter) - $(V_W(d)/E_A)/(V_W/E_A)$.

$V_W(d)_R$ - Voltage induced in reference one-foot-square, single-turn loop antenna submerged in water to a depth of d feet.

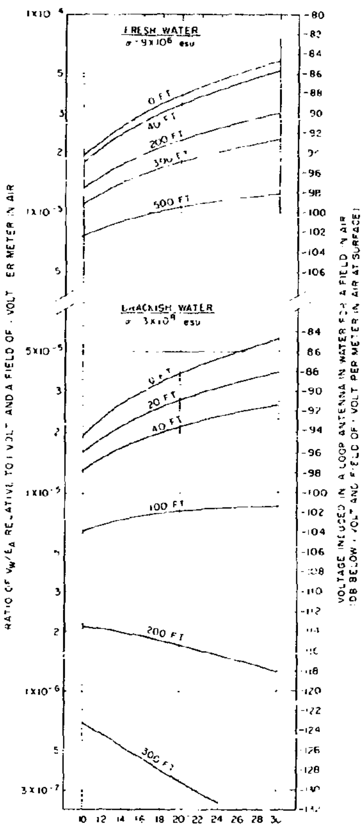
E_A - Electric field strength in air at the surface directly above the submerged loop.

$(V_W/E_A)_0$ - Refers to a convenient arbitrary reference of induced voltage with in-water operation of one volt and an electric field of one volt/meter in air

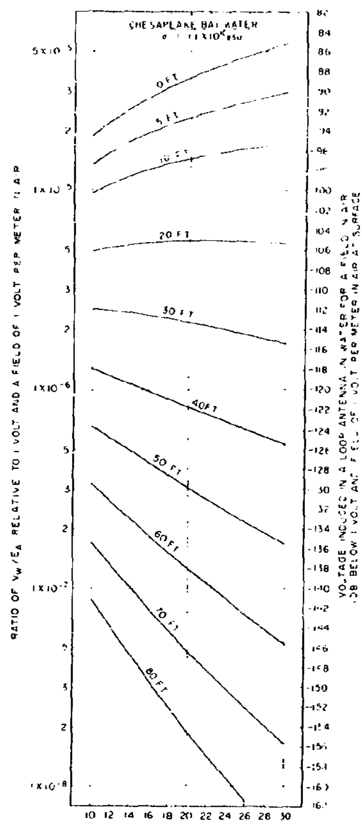
Perhaps at this point it is pertinent to explain the purpose and underlying reasons for introducing the three different unit reference factors, $(V_A/E_A)_o$, $(V_W/E_W)_o$, $(V_W/E_A)_o$. In each case a normalizing reference was introduced, so that the quantities to be dealt with would be dimensionless and therefore suitable for logarithmic transformation into more convenient computational forms. Unit reference factors were arbitrarily chosen simply for numerical convenience. Finally, it was necessary to introduce different reference factors in each case so that certain quantities could be properly expressed in decibels, as originally defined (i.e., ten times the common logarithm of a power ratio, or twenty times the common logarithm of a voltage ratio when the voltages appear across impedances of the same value). Although, for example, $(V_A/E_A)_o$ and $(V_W/E_A)_o$ are dimensionally the same, they are functionally different, since V_A refers to a voltage induced into an antenna in the air and V_W refers to a voltage induced into the antenna submerged in water - the essential difference being that in general the equivalent source impedance of the antenna when in air might be quite different from what it would be in another medium, such as sea water (however, since the impedance has been assumed to remain the same in this study, the distinction may perhaps seem unnecessary, except insofar as it helps to clarify the interpretation).

Figure 8a shows the effects caused by water of different conductivity on the induced loop-antenna voltage for a given field strength in air relative to that of the reference one-foot-square single-turn loop R over the restricted frequency range of 10 to 40 kc for the loop at zero depth of submergence (i.e., with the top of the loop just under the water surface). It will be noticed that the induced voltage increases considerably as the frequency increases (for the zero-submergence case), but that it decreases only slightly as conductivity increases. However, as shown in Fig. 8b, the situation is considerably changed at a 20-foot depth of submergence, for in this case the induced voltage is considerably reduced by operation in sea water, and the previous rising trend of induced loop voltage is reversed with frequency increase. It is evident that increasing the depth of loop submergence in water increases the separation between the induced voltages for the four values of water conductivity illustrated in these graphs. There is an apparent advantage in favor of higher frequency operation as the water becomes fresher. It is interesting to note that for loop antennas submerged in Chesapeake Bay water at this particular depth, the response is fairly insensitive to frequency changes within the range shown. It would appear that such a flat frequency-response condition might occur at some decreased depth in sea water, also. This is shown to be the case in Fig. 9, where it appears that with a ten-foot depth of submergence, the induced voltage is relatively constant (variation is within about 1 decibel) over the frequency range between about 10 and 40 kc. Figure 9 gives information over a considerably wider range of frequency (0.1 to 1000 kc) and for increased depths of submergence (down to 500 feet) in sea water. Response peaks occur at an increasingly lower frequency with increase in the depth of operation. (A derivation of the frequency which gives peak response at a given depth is given in Appendix F.) Figure 9 graphically shows the importance of operating at the lower frequencies when operation at considerable depth is desired, but it also indicates that certain advantages might be obtained at the higher frequencies if shallow loop operation should be feasible. Of course, the increased propagation loss in air at the higher frequencies (Fig. 9) has not been taken into account, since Fig. 9 assumes a fixed amount of field strength in air at the surface, immediately above the submerged loop antenna. However, in situations where the in-air propagation attenuation is not a prime consideration (e.g., where the transmitter is in the immediate vicinity of a submerged antenna), Fig. 9 is useful for determining the field in air necessary for producing some desired amount of induced voltage in a submerged water-core rectangular loop antenna at some particular depth in sea water.

Figures 10a and 10b show the voltage induced in a loop antenna submerged in fresh, brackish, and Chesapeake Bay water, respectively, over the part of the vlf range normally

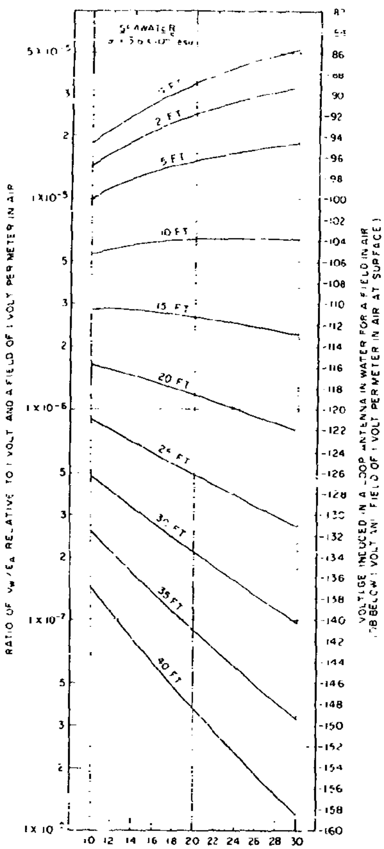


(a) Voltage induced in reference loop antenna submerged in fresh and brackish water

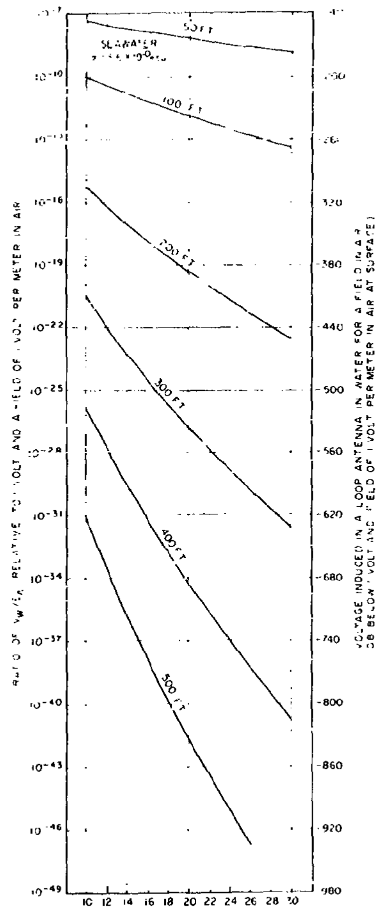


(b) Voltage induced in reference loop antenna submerged in Chesapeake Bay water

Fig. 10 - Effect of depth on the voltage induced in reference loop antenna submerged in several types of water with a given field strength in air as a function of frequency (with respect to an induced voltage of 1 volt and a field strength in air of one volt per meter)



(c) Voltage induced in reference loop antenna submerged to normal depths in sea water



(d) Voltage induced in reference loop antenna deeply submerged in sea water

Fig. 10 (Continued) - Effect of depth on the voltage induced in reference loop antenna submerged in several types of water with a given field strength in air as a function of frequency (with respect to an induced voltage of 1 volt and a field strength in air of one volt per meter)

of interest for submerged-communications reception. Figures 10c and 10d give essentially the same sort of information as shown in Fig. 9 for sea water, except in somewhat greater detail, on an expanded scale restricted to the vlf range and with respect to the same linear frequency scale used for Figs. 10a and 10b.

Voltage Interface Loss

The loss in induced voltage sustained by a loop antenna in moving from a position entirely in air just above the surface to a position entirely in water but with the top of the loop just under the water surface is termed the voltage-interface loss and may be expressed as the ratio $V_W(0)/V_A$. This ratio indicates directly the amount of loss which must be compensated by an increase in field strength at a given range to allow submerged operation just beneath the surface.

Another very useful identity, similar to that given by Eq. (23), can now be established which relates this ratio to several of the system elements already discussed:

$$\frac{V_W(d)}{E_A} = \overset{\text{(Voltage-interface loss)}}{\frac{V_W(0)}{V_A}} \times \overset{\text{(Depth-of-submergence loss)}}{\frac{V_W(d)}{V_W(0)}} \times \overset{\text{(Antenna collection capability in air)}}{\frac{V_A}{E_A}} . \tag{27}$$

Thus the voltage induced in a submerged loop antenna at a specified depth for a given radio field intensity in air is equal to either the product of the field-interface loss, the depth-of-submergence loss, and the loop-collection capability in water (Eq. 23), or to the product of the voltage-interface loss, the depth-of-submergence loss, and the loop collection capability in air (Eq. 27).

Since the voltage induced in a submerged loop is directly proportional to the field intensity at the top of the loop (as indicated by Eq. 13), the depth-of-submergence loss expressed in terms of the reduction in induced voltage is the same as that expressed in terms of the loss in field strength which occurs with increased depth; i.e.,

$$\frac{V_W(d)}{V_W(0)} = \frac{E_W(d)}{E_W(0)} = \epsilon^{-\frac{d}{\delta}} . \tag{28}$$

Equating Eqs. (23) and (27), and solving for the voltage-interface loss using the equality established by Eq. (28), gives

$$\frac{V_W(0)}{V_A} = \frac{\left(\frac{V_W(d)}{E_W(d)}\right)}{\left(\frac{V_A}{E_A}\right)} \times \left(\frac{E_W(0)}{E_A}\right) . \tag{29}$$

or

$$\overset{\text{(Voltage-interface loss)}}{\frac{V_W(0)}{V_A}} = \overset{\text{(Antenna pickup gain with operation in water)}}{G} \times \overset{\text{(Field-interface loss)}}{\left(\frac{E_W(0)}{E_A}\right)} . \tag{30}$$

where

$$G = \frac{\left(\frac{V_W}{E_W}\right)}{\left(\frac{V_A}{E_A}\right)} = \frac{\left(\frac{V_{WR}}{E_W}\right) \times F_{CW}}{\left(\frac{V_{AR}}{E_A}\right) \times F_{CA}} = G_R \times \frac{F_{CW}}{F_{CA}} . \tag{31}$$

The ratio of the loop-antenna collection capability in water to that in air is referred to as G, the loop-antenna collection capability improvement, or gain, with operation in water as compared to in-air operation. (Equation 30 indicates that this gain is also equal to the ratio of the voltage-interface loss to the field-interface loss.) A comparison of the curves of Fig. 7a with the curve in Fig. 6a shows that the signal voltage induced in the reference-loop antenna in water is indeed greater than that for the same loop in air for equivalent field intensities in both air and water. The curves of Fig. 11 show G_R, the improvement in collection capability with operation in water for the reference loop antenna, plotted over the 10- to 1000-kc frequency range and for the water-conductivity values previously employed. Th data represent the difference (in decibels) between those shown in Fig. 7a for water and those in Fig. 6a for air. An examination of Fig. 11 shows that the improvement in reference loop-antenna collection capability upon its submergence in water is considerable for equal field intensities in both mediums. For example, the gains with reference-loop operation in sea water are 2510, 1750, and 1190 times at 10, 20, and 40 kc, respectively. Thus, since Eq. (30) indicates that the voltage-interface loss is the product of the gain and the field-interface loss, it appears that the large improvement in loop-antenna collection capability with operation in water will compensate in large measure for the considerable loss in field intensity in transiting the air-to-water interface. Therefore the voltage-interface loss should normally not be expected to be very large. Figure 11 also shows that the pickup gain, or improvement, for the reference loop decreases as the signal frequency increases and/or the water conductivity decreases. Equation (31) indicates that the improvement in pickup for other sizes of loops may be obtained simply by multiplying the gain for the reference-loop antenna by the ratio of F_{CW} to F_{CA} (Eq. 34).

The voltage-interface loss for any given loop antenna may be expressed in terms of the reference loop R by defining a factor F_{VIL} as follows:

$$F_{VIL} \times \left(\frac{V_W(0)}{V_A}\right)_R = \frac{V_W(0)}{V_A}, \text{ hence } F_{VIL} = \frac{V_W(0)\ V_A}{V_W(0)_R\ V_{AR}}. \tag{32}$$

By substituting Eq. (9) into Eq. (27) and employing the F_{VIL} factor, we have

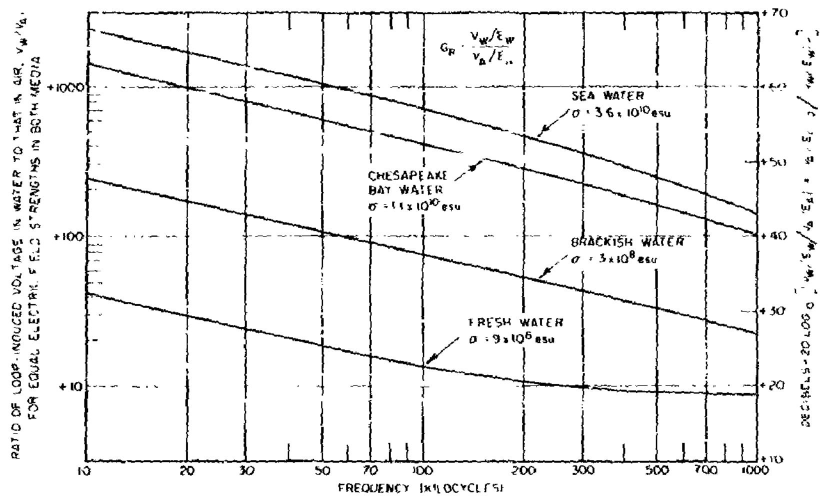


Fig. 11 - Reference-loop-antenna collection-capability improvement with operation in water for several salinity conditions as a function of frequency

$$\frac{V_W(d)}{E_A} = \left(\frac{V_W(0)_R}{V_{AR}} \times F_{VIL}\right) \times \left(\frac{V_W(d)}{V_W(0)}\right) \times \left(\frac{V_{AR}}{E_A} \times F_{CA}\right) . \tag{33}$$

Comparing Eq. (33) with Eq. (25) leads to the conclusion that

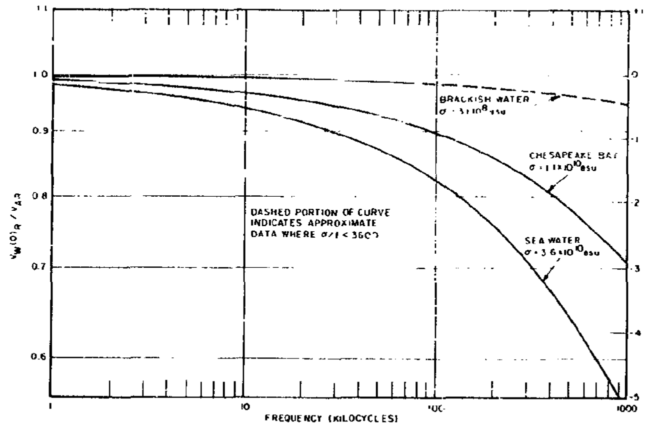
$$F_{CW} = F_{VIL} \times F_{CA} , \quad \text{or} \quad F_{VIL} = \frac{F_{CW}}{F_{CA}} . \tag{34}$$

Combining the above result with Eqs. (30), (31), and (32) yields a useful expression for the voltage-interface loss ratio for the reference loop:

$$\frac{V_W(0)_R}{V_{AR}} = G_R \times \frac{E_W(0)}{E_A} . \tag{35}$$

Figure 12a shows the voltage-interface loss ratio for the reference loop, as computed in accordance with Eq. (35), for three conditions of water conductivity and over the same 10- to 1000-kc frequency range previously employed for the field-interface loss curves in Fig. 4.[14] An examination of Fig. 12a reveals that unlike the field curves of Fig. 4, the reference voltage-interface loss increases as the frequency increases. Also very evident is the much lower magnitude of the voltage-interface loss ratio as compared to the

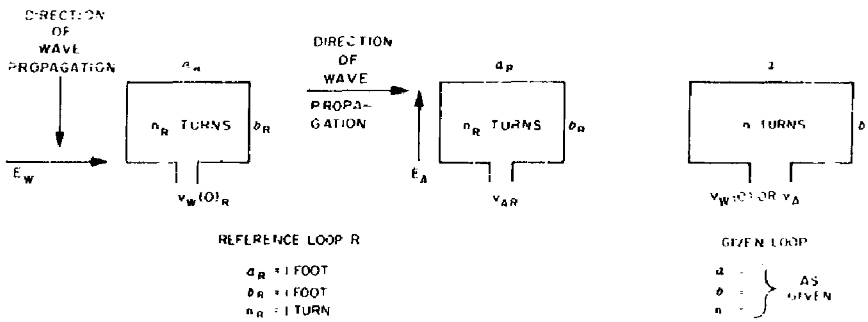
[14]The curves for brackish water in Fig. 12a are shown dashed beyond about 70 kc because the field-interface loss equation (Eq. 4) used in computing voltage-interface loss has been applied beyond its specified range of validity.



(a) Reference-loop-antenna voltage-interface loss for several salinity conditions for reference one-foot-square, single-turn loop R as a function of frequency.

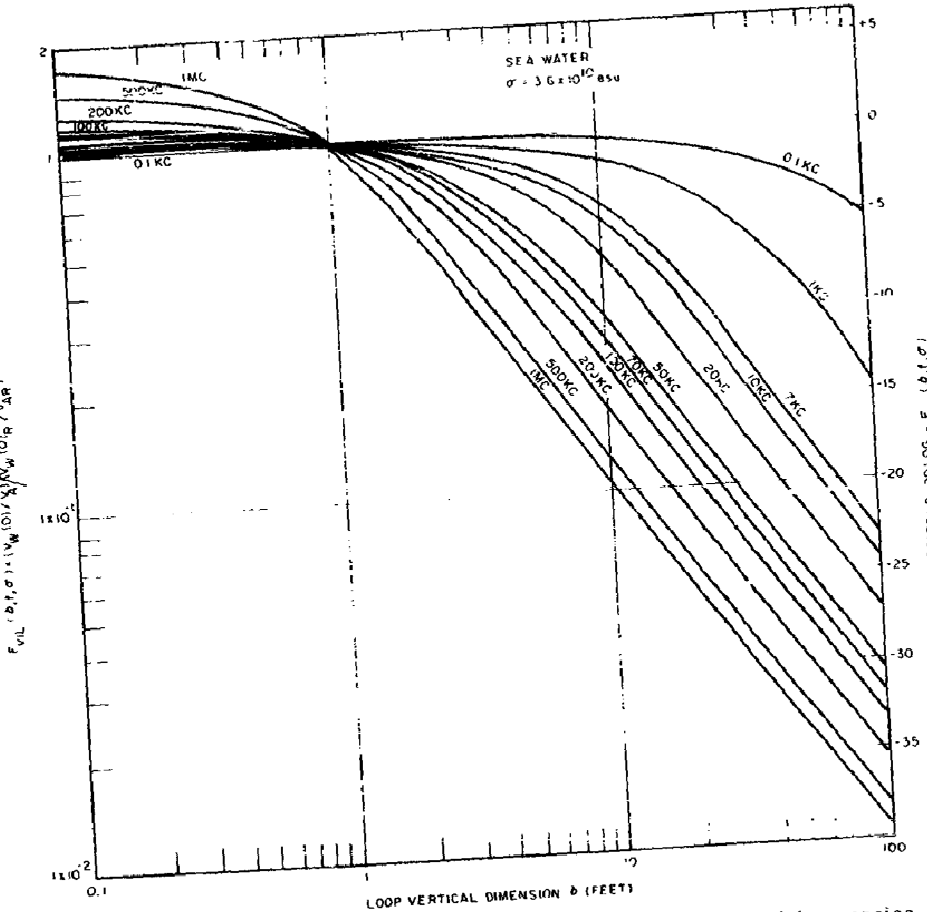
V_{AR} - Induced voltage with reference loop in air just above surface.
$V_W(0)_R$ - Indicates top of submerged loop at water surface, i.e., induced voltage with loop at zero depth.

Fig. 12 - Loop-antenna voltage-interface loss - induced (or open-circuit terminal) voltage with loop just submerged in water relative to that with loop in air for a given field strength in air



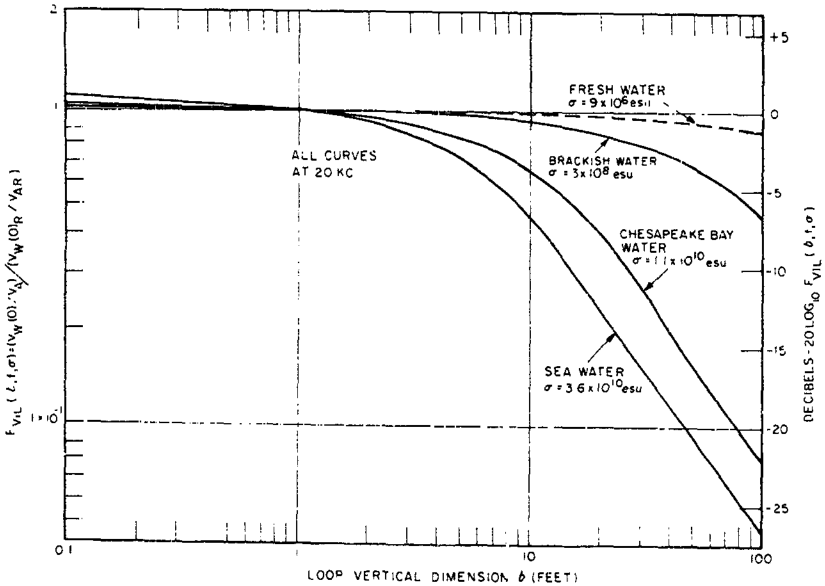
(b) Extension of the treatment of voltage-interface loss to include loops with dimensions and number of turns different from those for the reference loop. Note: Inductance and Q have not been required to remain fixed with change of a, b, and n relative to reference loop R.

Fig. 12 (Continued) - Loop-antenna voltage-interface loss - induced (or open-circuit terminal) voltage with loop just submerged in water relative to that with loop in air for a given field strength in air



(c) Relative loop-antenna voltage-interface loss in sea water for several frequencies as a function of the loop vertical dimension. Note: Inductance and Q have not been required to remain fixed with change of a, b, and n relative to reference loop R.

Fig. ... (Continued) - Loop-antenna voltage-interface loss - induced (or open-circuit terminal) voltage with loop just submerged in water relative to that with loop in air for a given field strength in air



(d) Relative loop-antenna voltage-interface loss at 20 kc for several salinity conditions as a function of the loop vertical dimension

Fig. 12 (Continued) - Loop-antenna voltage-interface loss - induced (o: open-circuit terminal voltage with loop just submerged in water relative to that with loop in air for a given field strength in air

field-interface loss ratio. For example, with a given field strength in air, the loop open-circuit terminal voltage in sea water at 10, 20, and 40 kc is 0.540, 0.817, and 0.886 of the voltage for the same loop in air, whereas the field intensity in sea water is only 3.72×10^{-4}, 5.25×10^{-4}, and 7.41×10^{-4} of the field in air, respectively.

Since the voltage-interface loss for the reference loop is rather small, being on the order of 10 percent or less in the vlf range, it is evident that the gain or improvement in loop-antenna collection capability with operation in water very nearly compensates for the large field-interface loss.

The voltage-interface loss for rectangular open-core loops with dimensions other than those of the reference loop may be calculated directly from the following expression (or from the data in Figs. 12c and 12d, using Eq. 32), which has been derived by expanding Eq. (29) in terms of the expressions developed for the analytical rectangular open-core loop-antenna model:

$$\frac{V_W(0)}{V_A} = \frac{a}{b}\sqrt{\left(\frac{\omega}{8\pi\sigma}\right)\left(\frac{1 - 2\epsilon^{\theta}\cos\theta + \epsilon^{2\theta}}{1-\cos\frac{\omega a}{c}}\right)}. \tag{36}$$

It is apparent that the voltage-interface loss is not a function of the number of turns in a loop. Therefore F_{VIL} must also be an independent function of n. An investigation of Eq. (36) as a function of loop dimension a has revealed that the voltage-interface loss is essentially independent of a wherever $fa \ll 5.4 \times 10^8$, since $\sqrt{2[1 - \cos(\omega a/c)]}$, which is then very nearly equal to $\omega a/c$ (as shown in Appendix C), in effect cancels the only other a factor in the expression. This, of course, results in F_{VIL} also being essentially independent of a (within the restrictions mentioned above). Expressing Eq. (32) in logarithmic form gives

$$\begin{aligned} 20\log_{10} F_{VIL}(a,b,f,\sigma) = {} & [F_{CW}(b,f,\sigma)]_{dbv}\Big|_{a_R n_R} \\ & - [F_{CA}(a,f)]_{dbv}\Big|_{b_R n_R} + 20\log_{10}\left(\frac{a}{a_R}\right) - 20\log_{10}\left(\frac{b}{b_R}\right). \end{aligned} \tag{37}$$

When $fa \ll 5.4 \times 10^8$,

$$20\log_{10} F_{VIL}(a,b,f,\sigma) \doteq 20\log_{10} F_{VIL}(b,f,\sigma). \tag{38}$$

Figure 12c shows $F_{VIL}(b, f, \sigma)$ plotted as a function of the b dimension for several frequencies between 0.1 and 1000 kc for the case of sea water. The curves show that a greater voltage-interface loss results with larger b loop dimensions. The loss is also greater with higher frequency for values of b greater than one.

The effects of variation in the water conductivity on the F_{VIL} factor at 20 kc is shown in Fig. 12d. It may be seen that there is little effect for loop antennas with b dimensions less than about one foot, but that the relative voltage-interface loss factor increases considerably as the conductivity approaches that of sea water for the larger b dimensions. Table 3 gives similar data for several other frequencies.

Figure 13 shows in a more direct fashion the difference in loop-antenna induced voltage for a given field strength in air between in-air and submerged operation. The voltage induced in the reference loop for a fixed field in air is shown plotted over the vlf

Table 3
Water Salinity Effects on F_{VIL} at Various Frequencies (Loop-Antenna Voltage-Interface Loss for Reference Antenna is Shown in Fig. 12a, 20-kc Data is Shown in Fig. 12d, F_{VIL} is Independent of n and a)

f (kc)	σ (esu)	F_{VIL} in dbv b = 0.1 Ft	b = 0.5 Ft	b = 1 Ft	b = 2 Ft	b = 5 Ft	b = 10 Ft	b = 20 Ft	b = 50 Ft	b = 100 Ft
1	σ'	+0.149	+0.083	0	-0.166	-0.666	-1.498	-3.159	-7.997	-14.370
1	σ''	+0.083	+0.046	0	-0.092	-0.368	-0.828	-1.748	-4.494	-8.872
1	σ'''	+0.014	+0.0076	0	-0.015	-0.061	-0.137	-0.289	-0.745	-1.504
1	σ''''*	+0.0023	+0.0013	0	-0.0026	-0.011	-0.024	-0.050	-0.129	-0.260
10	σ'	+0.474	+0.263	0	-0.526	-2.104	-4.712	-9.602	-18.154	-24.155
10	σ''	+0.262	+0.145	0	-0.291	-1.164	-2.616	-5.490	-12.927	-19.250
10	σ'''	+0.043	+0.024	0	-0.048	-0.192	-0.432	-0.913	-2.352	-4.737
10	σ''''*	+0.007	+0.004	0	-0.008	-0.033	-0.073	-0.154	-0.399	-0.807
50	σ'	+1.059	+0.588	0	-1.176	-4.668	-9.981	-16.539	-24.473	-30.494
50	σ''	+0.586	+0.325	0	-0.651	-2.598	-5.794	-11.471	-19.851	-25.869
50	σ'''	+0.096	+0.054	0	-0.107	-0.428	-0.964	-2.035	-5.226	-10.196
50	σ''''*	+0.015	+0.009	0	-0.017	-0.067	-0.150	-0.317	-0.823	-1.677
100	σ'	+1.276	+0.832	0	-1.661	-6.498	-12.872	-19.038	-26.996	-33.017
100	σ''	+0.528	+0.460	0	-0.920	-3.664	-8.041	-14.572	-22.588	-23.609
100	σ'''	+0.136	+0.076	0	-0.151	-0.604	-1.359	-2.869	-7.311	-13.490
100	σ''''*	+0.019	+0.013	0	-0.021	-0.085	-0.192	-0.407	-1.067	-2.214

NOTE: σ' = 3.6×10^{10} esu ≈ Sea water
σ'' = 1.1×10^{10} esu ≈ Chesapeake Bay water
σ''' = 3.0×10^{8} esu ≈ Brackish water
σ'''' = 9.0×10^{6} esu ≈ Fresh water

*See text in connection with discussion of Fig. 4.

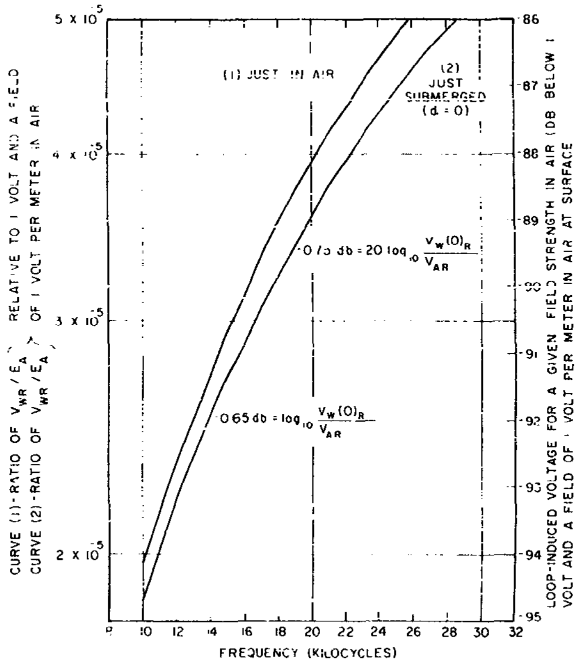


Fig. 13 - Voltage induced in reference one-foot-square, single-turn loop antenna for a unit field strength in air for (1) operation just in air at surface, and (2) operation just submerged in sea water

range from 10 to 30 kc for the just-submerged condition in sea water (i.e., $d = 0$, or with the top of the loop just below the sea surface) and the just-in-air condition (i.e., bottom of loop just above the surface). The difference between these two curves at any frequency shown corresponds, of course, to the voltage-interface loss ratio, $V_W(0)_R/V_{AR}$. The loop-induced voltage per unit of field strength is shown relative to one volt and a field strength in air of one volt per meter.[15]

Loss Compensation Necessary for Underwater Operation

For satisfactory performance with underwater loop operation, the field strength in the air immediately above the surface over the loop antenna must be sufficiently greater than that field which would be satisfactory for in-air loop operation to compensate for the

[15] The discussion on p. 34 with regard to the choice of references is particularly pertinent in the interpretation of Fig. 13.

voltage-interface and depth-of-submergence losses for whatever operational loop depth is required.[16] Equation (22) related the necessary field for satisfactory submerged-loop operation to the necessary loop-induced voltage for threshold sensitivity, and Eq. (20) related the necessary loop-induced voltage for this output to the in-air field sensitivity of the receiving system. Combining these equations leads to an interesting relation which, after establishing another identity, explains the basis for the above statement:

$$\underbrace{\frac{E_{A_0}}{E_n}}_{\text{(Additional loss with submerged operation)}} = \frac{V_W(d)}{V_A} = \underbrace{\frac{V_W(0)}{V_A}}_{\text{(Voltage-interface loss)}} \cdot \underbrace{\frac{V_W(d)}{V_W(0)}}_{\text{(Depth-of-submergence loss)}} \tag{39}$$

Being independent of receiving-system parameters, the depth-of-submergence loss may be rather easily determined and taken into account when totaling up and allowing for the various losses in the system. However, the voltage-interface loss for any particular receiving system must be known in order to relate properly the field strength in air which is required to provide design threshold performance with submerged operation to the in-air field sensitivity figure for the same receiving system.[16] Thus, at least two measurements are necessary for determining the performance capability of a receiving system for submerged operation: the in-air field sensitivity E_{A_0} and the voltage-interface loss $V_W(0)/V_A$. The voltage-interface loss is unfortunately a rather complicated function of the various parameters of the receiving system. Perhaps the most satisfactory method of determining the voltage-interface loss for an actual receiving system is to measure it experimentally by noting how much the induced voltage drops as the antenna is lowered to a position just beneath the surface.

An analytical expression for the voltage-interface loss may, of course, be used whenever a satisfactory one is available, for example the case of the expression (Eq. 36) developed for the rectangular open-core loop antenna. Hence, evaluating Eq. (39) for E_n, the radio field required in air for minimum satisfactory system output performance, for the rectangular open-core loop case, gives

$$E_n = \frac{b\, E_{A_0}}{a_{\cdot}\,\frac{\cdot}{6}} \sqrt{\left(\frac{R \cdots}{\cdot}\right)} \left(\frac{1-\cos(\cdot\, a\, c)}{1 - 2\epsilon^{\cdot}\cos \cdots + \epsilon^{2a}}\right). \tag{40}$$

System Equations for Submerged Operation

The expressions for the various elements of a vlf submerged communication system may now be combined in the same manner as was done in the case for surfaced operation. Using again the basic relation $E_A \geq E_s$, and substituting Eqs. (2) and (39), a system equation for submerged operation can now be established in terms of E_{A_0}, the threshold field sensitivity in air (measured or specified at the signal frequency) of the receiving system:

---

[16] The assumptions outlined on p. 28 are, of course, particularly pertinent here and should not be neglected, since they underly this discussion.

$$E_A \geq \frac{E_0}{l_s} = \frac{E_{A_0}}{l_s \left[\frac{V_W(0)}{V_A}\right]\left[\frac{V_W(d)}{V_W(0)}\right]}. \tag{41}$$

Using the modified Baldwin-McDowell empirical formula for E_A, and applying it to the case of the rectangular open-core loop antenna, gives

$$\frac{5.20 \times 10^{-3} \sqrt{P_r}}{D} \epsilon^{-1.3\times10^{-8} fD} \geq \frac{E_{A_0} b}{l_s n \epsilon^{\frac{\theta d}{b}}} \sqrt{\left(\frac{8 n \sigma}{\omega}\right)\left(\frac{1-\cos(\omega \Psi C)}{1 - 2\epsilon^{\theta} \cos \theta + \epsilon^{2\theta}}\right)}. \tag{42}$$

Substituting for E_{A_0} from Eq. (20) allows this relation also to be expressed conveniently in terms of the threshold voltage sensitivity of the receiving system; thus

$$E_A \geq \frac{V_0}{l_s \left[\frac{V_A}{E_A}\right]\left[\frac{V_W(0)}{V_A}\right]\left[\frac{V_W(d)}{V_W(0)}\right]} \tag{43}$$

or, expanding in terms of the analytical model,

$$\frac{5.16 \times 10^{-3} \sqrt{P_r}}{D} \epsilon^{-1.3\times10^{-8} fD} \geq \frac{V_0}{l_s n a k \epsilon^{\frac{\theta d}{b}} \sqrt{\left(\frac{f}{2\sigma}\right)(1 - 2\epsilon^{\theta} \cos \theta + \epsilon^{2\theta})}}. \tag{44}$$

The basic system equations, Eqs. (41) and (43), are stated in very general terms and contain all of the various elements that determine and relate to overall performance capability of almost any submerged system. The other system equations, Eqs. (42) and (44), are stated in more specific terms, because they contain all of the various parameters that determine the overall performance capability of a particular submerged system using a rectangular open-core loop antenna.

It is evident upon comparing Eqs. (17) and (41), and also Eqs. (19) and (43), that the principal factors limiting the underwater reception of radio signals and which make the equations for surfaced operation different are the voltage-interface loss and depth-of-submergence loss, $V_W(0)/V_A$ and $V_W(d)/V_W(0)$. In other words, if these losses could be made negligible the equations would become identical, since the voltage ratios would then be equal to unity. For surfaced operation, of course, these losses are negligible, and Eqs. (41) and (43) can be applied for in-air operation as well. Therefore, Eqs. (41) and (43) can be interpreted as general system statements which apply both for surfaced and submerged operation.

III. RANGE DETERMINATION

OPERATIONAL DEPTH AS A FUNCTION OF RANGE

The system equations which culminated the previous discussion can be solved for the maximum loop-antenna depth for reliable communication at a given range. Such end-of-range limits are usually of primary concern for naval operational planning. The end-of-range limit is here taken as that point of operation where $E_A = E_s$, or in other words, where the radio field produced is just equal to that required for minimum satisfactory system output, allowing for operational losses as may be necessary. In order to apply the equations to the determination of permissible depth of submergence in any specific practical situation where numerical results are desired, it is, of course, necessary to choose specific numerical values for the various other parameters in the equations. The parameters which must be specified are transmitter radiated power P_r, system miscellaneous loss factor l_s, water conductivity σ, loop dimensions a and b, number of turns n on the loop antenna, the range D, the signal frequency f, and either the receiving-system in-air field or voltage sensitivity E_{A_0} or V_0, respectively.

To illustrate the application of the system equations for determining design end of range, certain numerical values have been selected which are more or less typical of the current operational situation. The radiated power has been chosen as one megawatt, on the basis of an estimated radiated power capability for the new Maine vlf transmitter now under construction. For "ideal" system conditions, no allowance need be made for miscellaneous operational system losses, in which case $L_s = 0$ db (or $l_s = 1$). For more practical system conditions, however, a 15-db system loss has been found to approximate actual operating experience on the basis of scattered operational reports. A 5-db variation about this value might be expected in practice. Sea water (σ taken as 3.6×10^{10} esu) has been chosen for all range calculations. A 30-turn, one-foot-square open-core tuned loop antenna has been selected as roughly approximating the loop antenna currently in operational use. Range has been chosen to include the largest ever likely to be needed. A frequency of 20 kc has been chosen, in the middle of the vlf band, as being fairly representative of the entire band. A design-threshold field-sensitivity figure, i.e., with $(S/N)_0 = 0$ db, of 2.2 μv/m at 18.6 kc has also been selected as representative of the omnidirectional sensitivity which should be achievable with such a loop system when tuned to resonate at the signal frequency.[17] A voltage sensitivity of 2390 $\mu\mu$v has been determined as consistent with the 30-turn, one-foot-square, tuned loop antenna and a value of V_{AR}/E_A at 18.6 kc of -88.8 db (Fig. 6a).

Equation 41 may be restricted to the end-of-range case, rearranged, and expressed in decibels using a logarithmic transformation in the following manner (using Eqs. 3 and 6):

$$S_f - L_s - L_v - L_d = 0, \tag{45}$$

or

$$d = \frac{1}{\alpha} [S_f - L_s - L_v]. \tag{46}$$

[17] Note: A current development program (10) shows some promise of an approximate 4:1 improvement in receiving-system sensitivity with a ferrite-core omnidirectional loop antenna.

where

$$S_f \text{ represents } 20 \log_{10} \left(\frac{E_A}{E_{A_0}}\right).$$

and

$$L_v \text{ represents } -20 \log_{10} \left(\frac{V_w(0)}{V_A}\right).$$

Figure 14 shows a plot of permissible depth of loop-antenna submergence for design threshold sensitivity output as a function of range at 20 kc for several conditions of miscellaneous system loss as calculated using Eq. (46), which provides a convenient form for computation in conjunction with Figs. 3, 5, and 12a. For example: according to Fig. 3, E_A = 120 µv/m, or 34.72 db above 2.2 µv/m at 7000 naut mi, so that S_f = 34.72 db; the voltage-interface loss for sea water as indicated in Fig. 12a is only about 0.75 db; allowing a 20 db operational system loss leaves an excess of about 13.97 db (34.72 - 0.75 - 20) which can be allowed for depth loss; dividing this figure by 1.49, the value for α obtained from Fig. 5, gives a maximum allowed depth of about 9.4 ft for the conditions specified. A reduction of 10 db in transmitter radiated power, from 1000 to 100 kw, would reduce the depth capability of the system shown by about 6.7 ft at all ranges at 20 kc. Figure 14 directly indicates the advantage in range to be gained by loop-antenna operation fairly near the surface. It is shown, also, that on the basis of 15 db extra system loss, loop submergence to about 20 ft is feasible at a range slightly in excess of 4000 naut mi for propagation entirely over sea water from a transmitter radiating one megawatt for loop-system design threshold sensitivity output conditions, i.e., $(S/N_0) = 0$ db. Propagation paths which are partly over land and/or ice[18] can be expected to reduce seriously the feasible depth of loop submergence at this range, or the range for a given depth.

OPERATIONAL RANGE AS A FUNCTION OF FREQUENCY

The selection of the carrier frequency is a very important parameter in the design of an effective submerged-reception radio-communication system. The system equations can also be transformed and solved to give the maximum theoretical range for reliable communication at a given depth of operation as a function of frequency. Expressing Eq. (42) in this manner gives a transcendental equation with respect to range D:

$$D\epsilon^{1.3 \times 10^{-8} f D} = \frac{5.10 \times 10^{-3} \sqrt{P_r}\, l_s\, \epsilon^{\alpha d/b_0}}{E_{A_0} b} \sqrt{\left(\frac{\omega}{8\pi\sigma}\right) \left(\frac{1 - 2\epsilon^{-\theta}\cos\theta + \epsilon^{-2\theta}}{1-\cos(\omega h/c)}\right)}. \qquad (47)$$

[18] The effects on vlf field strength of propagation over ice are not so well charted as they are for propagation over land. More precise information in this regard should be available upon completion of the IGY program.

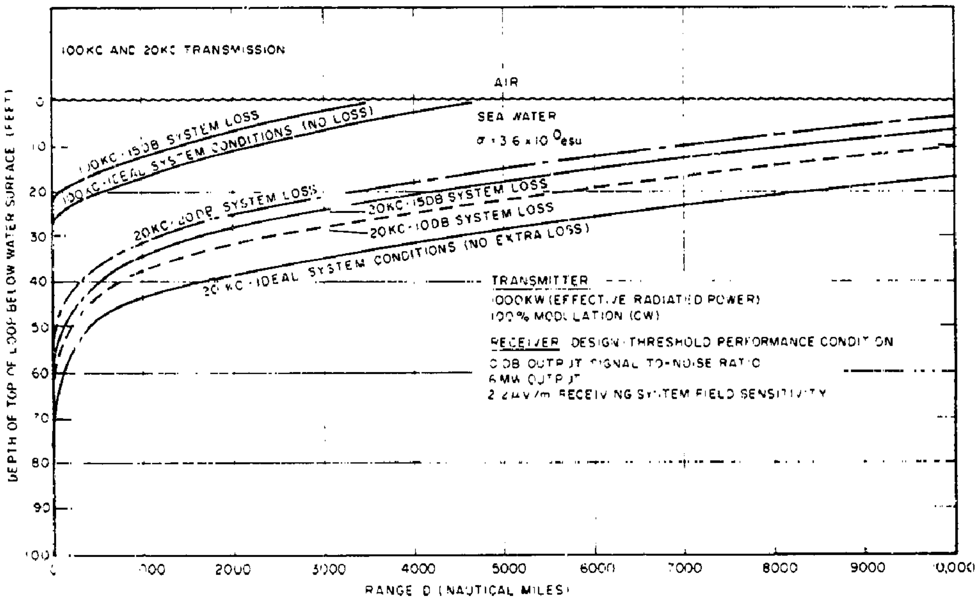


Fig. 14 - Theoretical depth capability vs range based on propagation entirely over sea water for an underwater radio receiving system employing a 30-turn, one-foot-square, open-core (water-core when submerged) loop antenna providing design threshold performance

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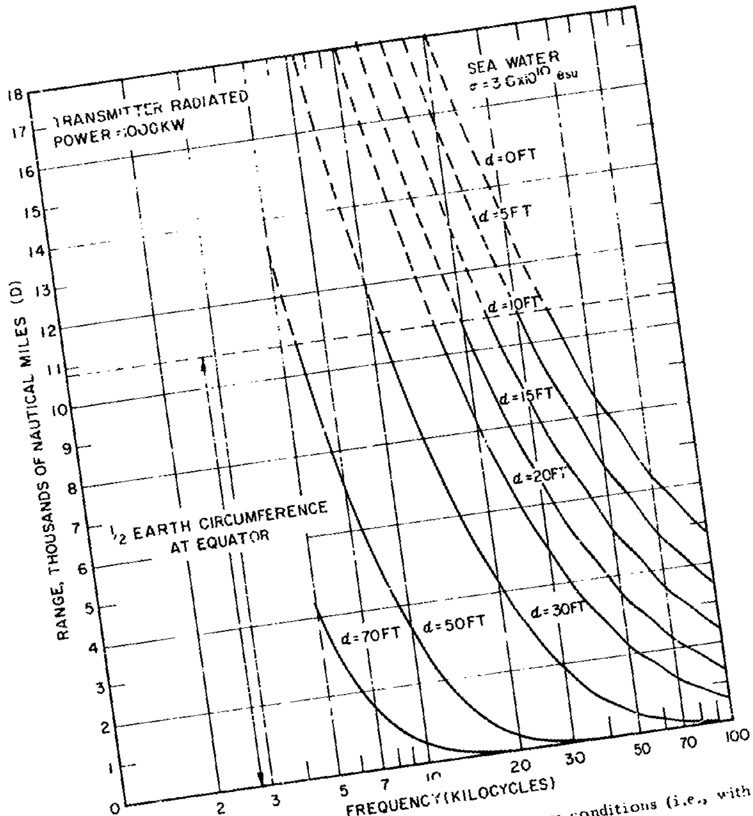
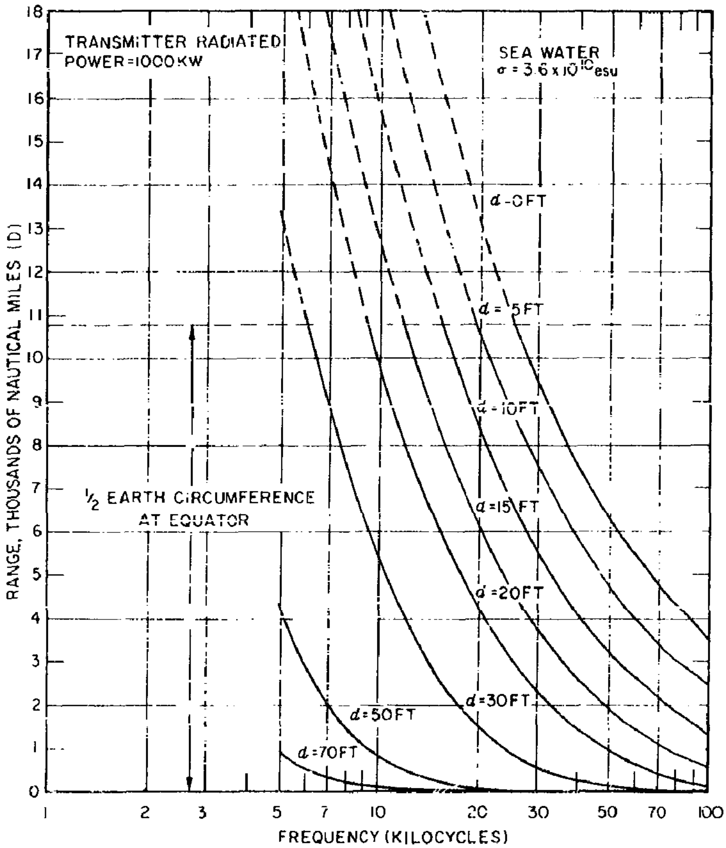


(a) Theoretical range vs frequency for ideal system conditions (i.e., with a 2.2-μv/m design threshold field sensitivity).

NOTE: The 2.2-μv/m field-sensitivity figure is probably practical only in the 18-kc frequency region

Fig. 15 - Theoretical range vs frequency at various depths of loop submergence based on propagation entirely over sea water and a fixed field-sensitivity condition for a loop receiving system employing a one-foot-square, open-core, 30-turn loop antenna



(b) Theoretical range vs frequency for 15-db system-loss conditions (which correspond to a 12.36-μv/m field sensitivity with no extra system loss).

NOTE: The 12.36-μv/m field-sensitivity figure is probably practical only in the 18-kc frequency region

Fig. 15 (Continued) - Theoretical range vs frequency at various depths of loop submergence based on propagation entirely over sea water and a fixed field-sensitivity condition for a loop receiving system employing a one-foot-square, open-core, 30-turn loop antenna

Such an equation is perhaps best solved using numerical techniques. Range has been computed using Newton's iterative method to solve this equation for frequencies extending from 5 to 100 kc. The same numerical values for the various parameters were used here as those previously selected for calculating the data for Fig. 14; these parameters are considered as being reasonably typical of the current operational situation. Figure 15a shows the values of range obtained for the "ideal" system case, and Fig. 15b shows those for the more practical 15-db extra-system-loss situation. The advantage of operation at the lower frequencies is quite apparent. The two principal causes of the decreased range at the higher frequencies are, of course, the increased propagation attenuation in the air and the increased depth-of-submergence loss.

Figure 16 shows the range for <u>surfaced</u> (in-air) operation as a function of frequency for both fixed field sensitivity and fixed voltage sensitivity and for both ideal and 15-db system-loss conditions. Comparing curve A on Fig. 16 with the $d = 0$ curve on Fig. 15a indicates that there is a rather slight loss of range with the loop antenna just submerged. This is consistent with the information presented in Fig. 13; i.e., the voltage-interface loss, which alone accounts for the loss in range with just-submerged operation, is not very large for a one-foot-square loop antenna. The equation used for computing <u>surface</u> range for a <u>fixed field</u> sensitivity is somewhat simpler than Eq. (47):

$$D\epsilon^{1.3 \times 10^{-8} f D} = 5.10 \times 10^{-3} \frac{\sqrt{P_r}}{E_{A_0}} l_s \tag{48}$$

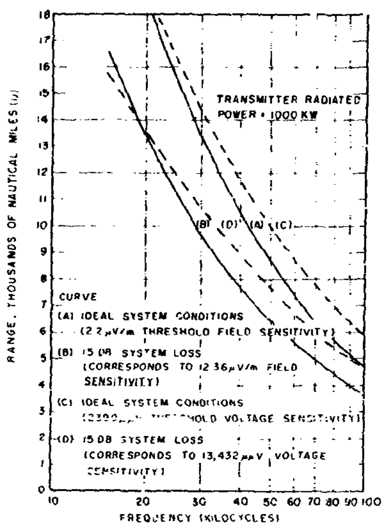


Fig. 16 - Theoretical range vs frequency for <u>surfaced</u> operation based on propagation entirely over sea water with a one-foot-square, open-core, 30-turn loop antenna showing both fixed field and fixed voltage sensitivity conditions

NOTE: 2.2-μv/m ≈ 2390 μμ v induced in a 30-turn loop at 18.6 kc; the 2.2-μv/m field-sensitivity figure is probably practical only in the 18-kc frequency range

It should be pointed out that the 2.2-μv/m -threshold field-sensitivity figure which was established at 18.6 kc is probably not strictly applicable across the large frequency range shown in Figs. 15, and 16, since the field sensitivity would normally be expected to become better with an increase in frequency. A fixed voltage sensitivity across the frequency range is probably more realistic. Curves C and D have thus been plotted on Fig. 16 for comparison with the fixed-field-sensitivity curves (A and B). The fixed-field-sensitivity curves intersect the fixed-voltage-sensitivity curves at about 18.6 kc, since it was at this frequency that the figure for voltage sensitivity was established to give output performance equivalent to that provided by a 2.2-μv/m radio field. The equation used for computing surface range for a fixed voltage sensitivity is based on Eq. (21), i.e.,

$$D \cdot 1.3 \times 10^{-8} f D \quad 5.10 \times 10^{-3} \frac{\sqrt{P_r}\, l_s}{V_0}\, n a b \quad \sqrt{2}\,\left(1-\cos(\omega a/c)\right) \cdot \qquad (49)$$

Using Eq. (44) as a basis, an equation for computing range for submerged operation with fixed voltage sensitivity can be expressed as

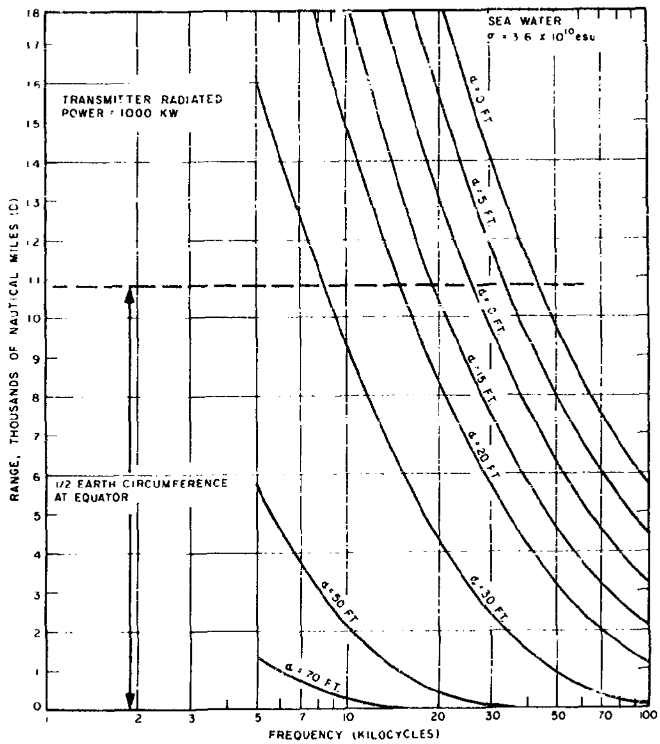
$$D \cdot 1.3 \times 10^{-8} f D \quad 5.10 \times 10^{-3} \frac{\sqrt{P_r}\, l_s}{V_0}\, n a k \epsilon^{-\alpha d} b \quad \sqrt{(f/20)(1-2\,\text{[illegible]}\cos\,\text{[illegible]}^{2})} \cdot \qquad (50)$$

Figures 17a and 17b show the theoretical range versus frequency with fixed voltage sensitivity at various depths of loop submergence for the same numerical values used previously for calculating the data for Figs. 14, 15a, 15b, and 16. Above 18.6 kc, the curves show increased range as compared to that shown in Fig. 15 for fixed field sensitivity. Figure 17 probably represents a more realistic appraisal of actual communication system capability and, therefore, should be the type used as a basis for frequency selection in system design.

THE RELATION BETWEEN LOOP-ANTENNA DIMENSIONS AND OPERATIONAL RANGE CAPABILITY

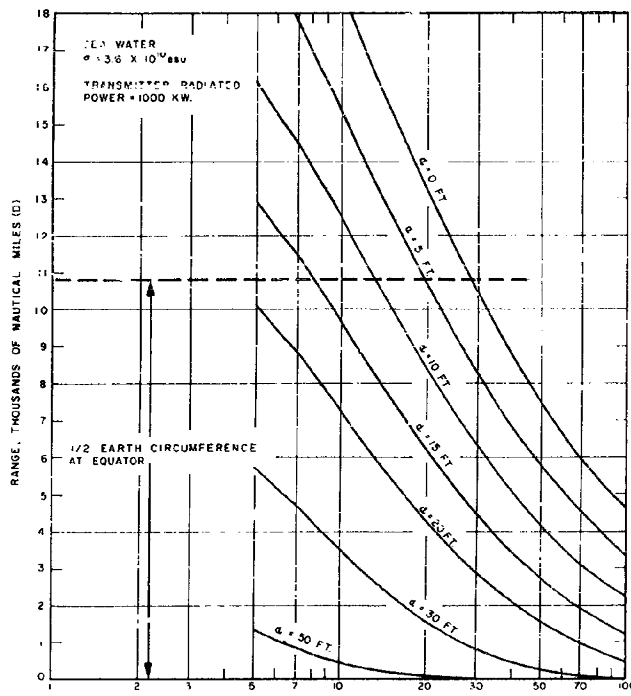
Since the system equations encompass all of the elements and parameters in a system, they provide a useful basis for determining how various parameters can affect range capability. The relation between the size of the loop antenna and range capability is of particular interest. Equation (47) is already expressed in a useful form for studying the effects on range caused by different loop dimensions for fixed field-sensitivity conditions.

It should be realized that the condition of fixed field sensitivity is normally not a very practical condition, since the figure for field sensitivity will usually change considerably with a change in loop dimensions. Figure 18 has nevertheless been plotted to show how end-of-range capability would vary as a function of frequency for loop antennas of three different vertical heights with respect to a fixed threshold field sensitivity (as measured in air) of 2.2 μv/m at all frequencies and for all three loops. In the practical sense, such a field-sensitivity value is most likely to be applicable for loops about one foot square in the 18-kc frequency region. However, the figure does, of course, give valid information wherever a 2.2-μv/m -threshold field sensitivity is applicable, and it may therefore be used to this limited extent for operational prediction. The decreased range capability indicated for the larger loops is caused by the increased voltage-interface loss which occurs with the larger b dimensions. The figure indicates that range based on a fixed field sensitivity is independent of the loop horizontal length, or a dimension. This follows



(a) Theoretical range vs frequency for ideal system conditions (i.e., with a 2390-$\mu\mu$v design threshold voltage sensitivity). 2390$\mu\mu$v $\approx$ 2.2μv/m at 18.6 kc with no extra system loss.

Fig. 17 - Theoretical range vs frequency at various depths of loop submergence based on propagation entirely over sea water and a fixed voltage-sensitivity condition for a loop receiving system employing a one-foot-square, open-core, 30-turn loop antenna



(b) Theoretical range vs frequency for 15-db system-loss conditions (which correspond to a 13,432-μμv voltage sensitivity with no extra system loss). 13,432 μμv ≈ 12.36 μv/m field strength at 18.6 kc, which represents 15-db system loss relative to ideal threshold sensitivity conditions.

Fig. 17 (Continued) - Theoretical range vs frequency at various depths of loop submergence based on propagation entirely over sea water and a fixed voltage-sensitivity condition for a loop receiving system employing a one-foot-square, open-core, 30-turn loop antenna

since the voltage-interface loss is essentially independent of changes in the a dimension. Of course, loop antennas with larger a dimensions usually have increased field sensitivity. Keeping field sensitivity fixed with respect to frequency and antenna dimensional changes (Fig. 18) is unrealistic for at least three reasons: (a) field sensitivity normally improves with an increase in frequency, (b) loop-antenna collection capability also improves, within limits, with an increase in the loop-antenna dimensions, and (c) any increase in loop-antenna dimensions can result in an increase in loop equivalent source impedance which, in turn, will tend to decrease the available power. The curves in Fig. 18 present factual (but not necessarily practical) information which shows the effect which various loop dimensions can have on overall system range capability. It should be understood that except for their effect on voltage-interface loss (which has been explicitly taken care of in the calculations), both the loop's physical dimensions and electrical design parameters, such as impedance and Q, are automatically taken into account in the overall system analysis when the receiving-system field sensitivity in air is measured or specified.

A plot with fixed voltage sensitivity is more indicative of actual operational range capability and therefore is much more useful for system design purposes. It is possible, however, that a fixed voltage sensitivity will be difficult to maintain as the loop-antenna dimensions are increased, due to a probable increase in loop equivalent source impedance and a consequent probable decrease in the available power from the loop. It is, of course, being assumed that the loop source impedance does not change due to antenna submergence; otherwise, this effect should as well be considered as a limitation when interpreting such a plot. Thus, a plot representing the culmination of many of the ideas developed in this report is shown in Figs. 19 and 20 for a fixed threshold voltage sensitivity of 2390 $\mu\mu$v, as as calculated using Eq. (50). Figure 19 shows how range capability is affected by changes in the b dimension, and effects due to changes in the a dimension are shown in Fig. 20. The advantage indicated for the larger loops is quite apparent.

DISCUSSION OF VLF SYSTEM PARAMETERS IN PERSPECTIVE

It must be kept in mind that the loop-antenna electrical design parameters which provide optimum overall system sensitivity are not necessarily compatible with the loop-antenna dimensions which provide maximum radio signal pickup. The amount of power available from a loop antenna also depends on the loop impedance, which is a function of the number of turns, wire size and material, shape and size of the loop, core material, and core losses (if any), and to some extent the characteristics of the medium, depending, of course, upon the degree of isolation afforded by insulation between the loop and the medium. Similarly, the environmental and operational requirements of the submarine place very practical restrictions on the dimensional parameters of submerged-loop antenna systems, and these may compromise both the physical and electrical aspects of system design.[19] However, each of the effects of electrical design, dimension change, and operational factors with regard to loop-antenna performance need to be determined and appraised separately before they can be properly evaluated in combination. The determination of the effect of a change in rectangular-loop-antenna dimensions upon a loop antenna's collection capability in air and in water, upon the voltage-interface loss, and finally upon the system range capability, is consistent with this method of attacking

[19] The compromises involved in the common optimization of all these elements and parameters with respect to maximum loop-antenna submerged-reception capability are not given a detailed treatment in this report, but they form a basis for continuing studies aimed at improving the submerged-reception capability of vlf radio systems.

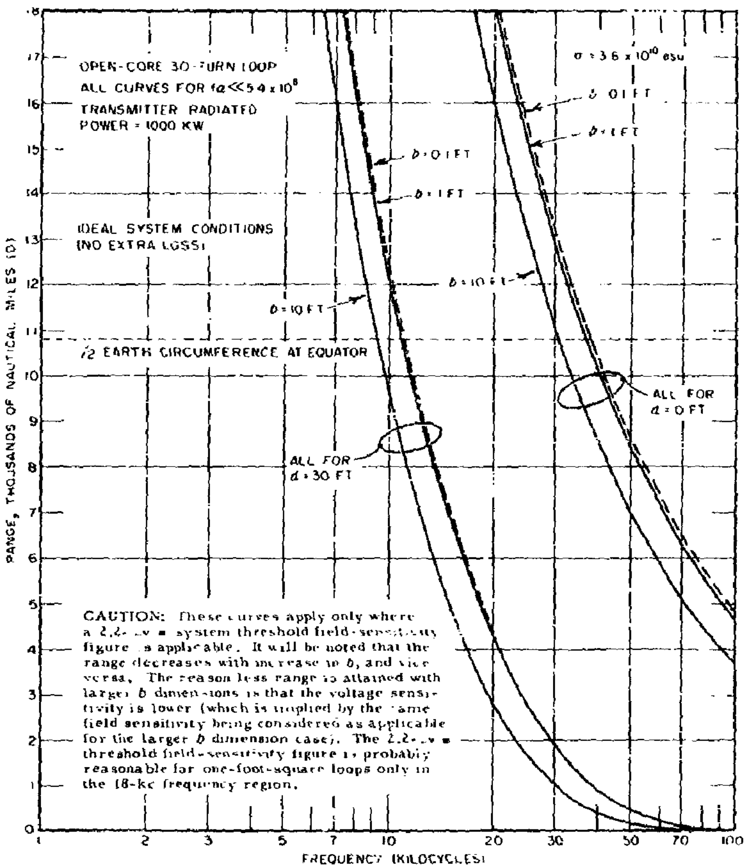


Fig. 18 - Range obtainable based on propagation entirely over sea water with receiving systems having different loop vertical dimensions and with a fixed 2.2-μv/m design threshold field sensitivity

NOTE: Inductance and C have not been required to remain fixed with change of a, b, or n relative to reference loop R

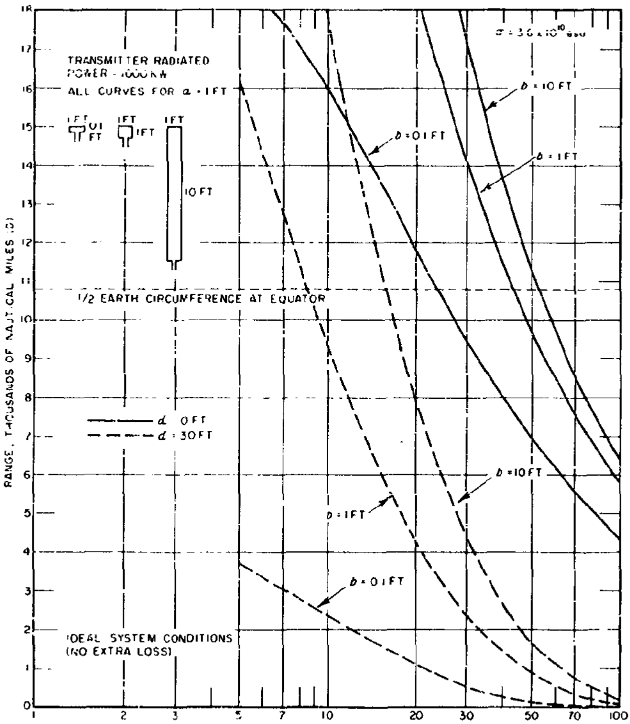


Fig. 19 - Range obtainable based on propagation entirely over sea water with receiving systems having different loop vertical dimensions but with a fixed 2390-$\mu\mu$v design threshold voltage sensitivity. 2390 $\mu\mu$v ≈ 2.2 μv/m at 18.6 kc with no extra system loss.

NOTE: Inductance and Q have not been required to remain fixed with change of a, b, or n relative to reference loop R

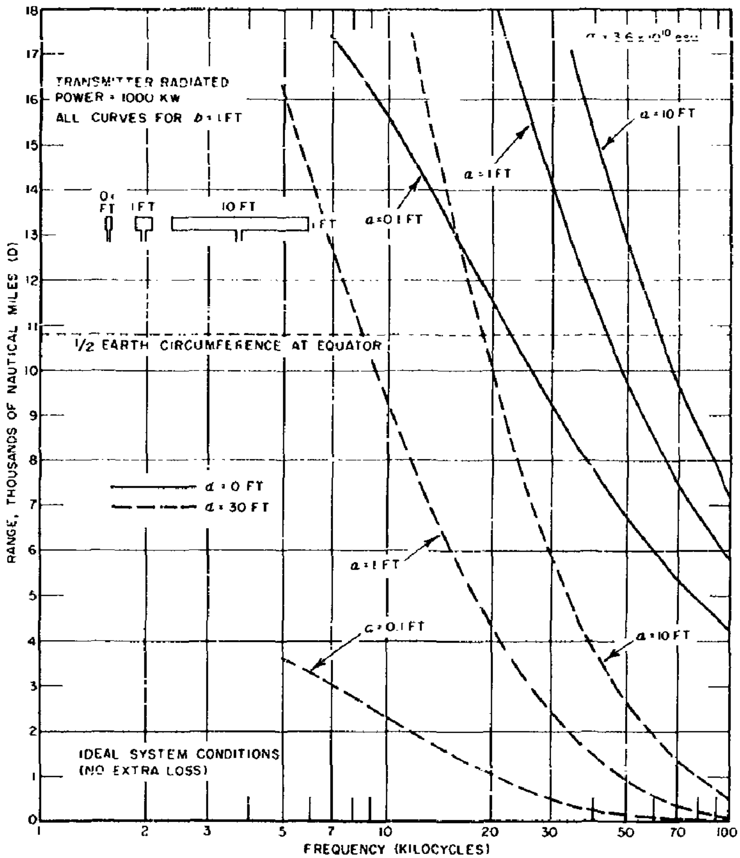


Fig. 20 - Range obtainable based on propagation entirely over sea water with receiving systems having different loop horizontal dimensions and a fixed 2390-$\mu\mu$v design threshold voltage sensitivity. 2390 $\mu\mu$v $\approx$ 2.2 $\mu\mu$ v/m at 18.6 kc with no extra system loss.

NOTE: Inductance and Q have not been required to remain fixed with change of a, b, or n relative to reference loop R

the overall problem, and it represents a first step toward the determination of the "optimum" (see footnote 19) antenna design consistent with maximum overall system-performance capability.

Unfortunately, the analytical treatment of submerged-loop antennas with core materials other than the surrounding medium is rather formidable, and apparently a comprehensive, thorough study of the case for a sea-water medium has not yet been made.[20] Furthermore, there is little experimental data available regarding the characteristics of submerged iron-core loops, largely due to the difficulty of making careful underwater measurements. Fratianni (5) has made experiments comparing an iron-core and an air-core loop both in water and in air, with the general conclusion being that the improvement obtained by an iron-core loop in water over an encased air-core loop in water is also realized when both loops are operated in air. However, reliable comparative data is needed to determine whether iron-core operation is superior or inferior to water-core operation. Theoretically, at least, it certainly is reasonable to expect that iron-core operation does not give as great an improvement over water-core operation as it did when compared with air-core operation, because of the radio-wave attenuation afforded by the water core, which provides higher output from a submerged water-core loop compared to a submerged air-core loop. A type of core material that retains the attenuation characteristic of water-core loops but which also offers a lower reluctance path to the field (a characteristic of most iron-core loops) might possibly prove to give better results than either iron- or water-core loops. A careful theoretical study of the optimum material and shape for a loop core is needed.

It should be realized that many of the loop antennas currently used in the U.S. Navy for submerged communications have iron cores and are not the open-core type for which Norgorden's expression and his equations, Eqs. (10) and (11), are intended. The justification for treating the water-core case in this report (other than the academic need to present a complete system picture showing how the various elements affect the final system result) is that range calculations made using equations applicable to the water-core case have predicted figures which have been in reasonably substantial agreement with scattered field reports of actual ranges attained using iron-core loops.[1] Furthermore, the application of modern weapon systems in conjunction with nuclear-powered submarines requires that antenna structures with satisfactory signal-collection capability be provided that permit deep submergence of the submarine while maintaining satisfactory communication. Iron-core loops occupying a volume much in excess of a cubic foot would probably be prohibited by the excessive weight of the iron, while larger water-core loops might be more feasible.

An examination of Norgorden's treatment (6) of the field in a conductive (sea-water) medium will indicate that he over simplified the problem by neglecting the effects of the dielectric constant of sea water. The inclusion of this effect into a more general and more complex treatment does not appear to lead to the simple plane-polarized waves he predicted. It would seem that perhaps a simple rectangular loop is not necessarily the best sort of device for coupling to a more complicated field. In a practical sense, however, the extra effects predicted by the more complex theory may very possibly be quite negligible, leaving an essentially plane-polarized wave as being the case at the lower frequencies. This question should certainly be resolved; a careful study of interface-refraction and underwater-propagation phenomena from the point of view of determining the exact nature of the field present and the most efficient coupling mechanism for extracting the utmost amount of energy from this field in the conducting medium is very desirable.

[20] Note, however, that the problem of radiation from a thin-wire loop antenna <u>in air</u> with a finite spherical core of material other than air has apparently been solved in a rigorous manner by Herman (11) and might serve as a suitable starting point for further work.

CONCLUSIONS

It is concluded that:

1. System equations can be established in terms of the elements of vlf radio-communication systems (e.g., propagation characteristics, interface loss), and applied to, for example, a system employing a rectangular, open-core loop antenna to depict system-performance capability in terms of basic independent parameters (such as frequency, water conductivity, and antenna dimensions), and to predict operational range capability, provided the characteristics of the elements are adequately described.

2. Relative air-to-water relationships involved in submerged radio reception can be established by identities which relate the basic system elements.

3. Loop-antenna pickup capability gain with operation in water nearly compensates for the large loss in radio field strength through the interface, falling short of full compensation by the amount of the voltage-interface loss, which for certain loop sizes and frequency conditions may not be negligible.

4. There is a particular loop-antenna vertical dimension corresponding to a given frequency which gives a maximum submerged pickup capability for a rectangular open-core loop; i.e., an increased vertical dimension beyond that will not result in further improvement and actually may cause a slight degradation.

5. The height dimension of a rectangular open-core loop antenna must be increased with a frequency decrease, if the maximum possible signal-voltage output with respect to changes in the loop height dimension is to be achieved.

6. The optimum loop-antenna vertical dimension increases as the water conductivity decreases.

7. An increase in the horizontal (width) dimension of a rectangular, open-core loop antenna is in general much more effective in increasing loop induced voltage than a corresponding increase in the loop vertical dimension.

8. An important concept evolving from this study is the fact that there is a particular frequency for a specified depth of loop-antenna submergence which gives a maximum submerged-loop pickup capability <u>for a fixed field strength in air</u>.

9. With a fixed field strength in air, the signal-voltage output from a small loop antenna increases with frequency in the vlf range at submerged depths down to about 10 ft in sea water, due to the fact that the increased loop pickup capability with increasing frequency overrides the increased depth-of-submergence loss at such shallow depths.

10. With a fixed field strength in air, the signal-voltage output from a small loop antenna usually decreases as frequency increases within the vlf range at submerged depths greater than about 10 ft in sea water, since the rate of depth-of-submergence-loss increase with increasing frequency is higher at the greater depths.

11. Systemwise, the apparent advantage of higher frequency operation at shallow depths for a specified field strength in air is lost, because with long-range operation the vlf propagation-attenuation characteristic in air is more favorable to the lower frequencies.

12. The overall range of vlf communications is improved by operation at the lower frequencies for the same amount of radiated power; however, there is an attendant increase in the wavelength and a decrease in the phase velocity of propagation in sea water as well as an increased cost of providing the same amount of radiated power with operation at lower frequency.

13. Expressing receiving-system performance in terms of the variations to be expected in basic system parameters with respect to a fixed voltage sensitivity is more indicative of realizable system performance capability than to a fixed field sensitivity.

14. The in-air field-sensitivity figure for the usual loop-type receiving system must be depreciated by the amount of the voltage-interface loss and the depth-of-submergence loss to yield the in-air field sensitivity figure for submerged operation at a given depth (assuming no change in loop source impedance with submergence).

15. Further theoretical studies are needed to determine the "optimum" vlf loop antenna design for both in-air and submerged reception, considering both optimum loop-antenna-system electrical design parameters and optimum loop dimensions which do not compromise the maneuverability, deep-submergence capability, or other operational requirements of the submarine.

RECOMMENDATIONS

It is recommended that additional theoretical studies and attendant experimentation be undertaken, leading to:

1. The "optimum" vlf loop antenna design for both in-air and submerged reception of electromagnetic waves, considering both optimum loop-antenna-system electrical design parameters and optimum loop shape and dimensions which do not compromise the operational requirements of the submarine in its water environment, and

2. A more efficient coupling mechanism to the available electromagnetic energy in a conducting medium than that afforded by a simple loop antenna.

ACKNOWLEDGMENTS

The help of Mr. E. Toth, Head of the Radio Techniques Branch, Naval Research Laboratory, for his many pertinent suggestions, based on extensive experience in this field, and for his constructive criticism is gratefully acknowledged. Thanks are due to Messrs. J. P. Falvey and B. O. Werle of this Branch for help in programming certain parts of the problem for the NAREC, and to Mr. F. M. Malone of CNO for helpful discussion in the initial stages of the study. Mr. S. V. Fratianni of the Radio Navigation Branch aided in preliminary general discussion leading to application of the basic work of the late Dr. O. Norgorden.

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APPENDIX A

DERIVATION OF THE MODIFIED BALDWIN-McDOWELL EMPIRICAL FIELD-STRENGTH FORMULA

The original Baldwin-McDowell empirical relation is expressed in the following form:

$$E_A = \frac{0.206\, Ihf}{D}\,\epsilon^{-0.000013 fD}\,, \tag{A1}$$

where the field strength E_A is expressed in μv/m, the antenna down-lead current I in amperes (as measured at the bottom of the down lead), the antenna effective height h in feet, the frequency f in kilocycles, the range D in nautical miles, and ϵ the base of natural logarithms. Now the radiated power P_r (in watts) is related directly to the antenna current by an expression which implicitly defines R_a, the radiation resistance of an antenna; viz.,

$$I = \sqrt{\frac{P_r}{R_a}}\ \text{amperes}. \tag{A2}$$

Various sources[1] give a very useful empirical relation for the realizable radiation resistance of a short vertical radiator over a good ground system, which may be expressed as

$$R_a = 1580 \left(\frac{hf}{c}\right)^2 \text{ohms}. \tag{A3}$$

The modified formula given in Eq. (1) of the text is obtained by substituting Eq. (A3) in Eq. (A2), substituting the resulting expression for the current in Eq. (A1) and simplifying. Thus,

$$E_A = 5.10 \times 10^{-3}\, \frac{\sqrt{P_r}}{D}\, \epsilon^{-1.3\times 10^{-8} fD}\ \text{volts/meter} \tag{A4}$$

where f is now in cps.

[1] For example, Eq. (A3) is given on p. 21 of the National Defense Contract (Nonr-2148(00)), "Final Engineering Report on Low and Very-Low-Frequency Antenna Study," Oct. 1957, by Federal Telecommunication Laboratories.

APPENDIX B

EQUATIONS FOR ELECTROMAGNETIC PLANE-WAVE PROPAGATION IN WATER[1]

Differentiating Maxwell's field equations with respect to time and substituting from one equation into the other gives the wave equations of the electromagnetic field, which, for the purpose at hand, may be simplified by assuming that $\mathbf{E}$ and $\mathbf{H}$ are functions of distance x and time t only:

$$\left.\begin{aligned} \frac{d^2\mathbf{E}}{dx^2} &= \epsilon^*\mu^*\frac{d^2\mathbf{E}}{dt^2}, \\ \frac{d^2\mathbf{H}}{dx^2} &= \epsilon^*\mu^*\frac{d^2\mathbf{H}}{dt^2}, \end{aligned}\right\} \tag{B1}$$

where

$\epsilon^* = \epsilon' - j\epsilon''$ and is defined as the complex permittivity of the medium, and

$\mu^* = \mu' - j\mu''$ and is defined as the complex permeability of the medium.

The solution of the differential equations with which we are concerned for the purposes of this report is a plane wave,

$$\left.\begin{aligned} \mathbf{E} &= \mathbf{E}_0\epsilon^{j\omega t - \gamma x}, \\ \mathbf{H} &= \mathbf{H}_0\epsilon^{j\omega t - \gamma x}, \end{aligned}\right\} \tag{B2}$$

advancing through the medium with a complex propagation factor,

$$\gamma = j\omega\sqrt{\epsilon^*\mu^*} = A + jB , \tag{B3}$$

where A is the attenuation factor and is the phase factor of the wave. Introducing these factors and substituting for , Eqs. (B2) may be rewritten in the form:

$$\left.\begin{aligned} \mathbf{E} &= \mathbf{E}_0\epsilon^{-Ax}\,\epsilon^{j2\pi\left[ft - \frac{Bx}{2\pi}\right]}, \\ \mathbf{H} &= \mathbf{H}_0\epsilon^{-Ax}\,\epsilon^{j2\pi\left[ft - \frac{Bx}{2\pi}\right]}, \end{aligned}\right\} \tag{B4}$$

from which it is apparent that the wave has a time period

[1] A. Von Hippel, "Dielectric Materials and Applications," New York: Wiley, 1954, discusses both the macroscopic and the molecular properties of nonmetallic materials. The derivation developed here is based principally on the theory treating the macroscopic aspects.

$$T = \frac{1}{f} . \tag{B5}$$

and a space period

$$\lambda = \frac{2\pi}{B} . \tag{B6}$$

The amplitude decays exponentially at a rate

$$\alpha = 20 \; A \; \log_{10} \epsilon , \tag{B7}$$

expressed in decibels per unit distance.

Surfaces of constant phase are given by

$$ft - \frac{x}{\lambda} = \text{constant} , \tag{B8}$$

which propagate with a phase velocity

$$\frac{dx}{dt} = v = f\lambda = \frac{\omega}{B} . \tag{B9}$$

Now, the complex relative permittivity and permeability may be defined, respectively, as:

$$\kappa^* = \frac{\epsilon^*}{\epsilon_0} = \kappa' - j\kappa'' ,$$

and (B10)

$$\kappa_m^* = \frac{\mu^*}{\mu_0} = \kappa_m' - j\kappa_m'' .$$

For a nonmagnetic medium such as sea water, the complex relative permeability simplifies to

$$\kappa_m^* = \kappa_m' \doteq 1 . \tag{B11}$$

The product of angular frequency and the relative complex permittivity loss factor is proportional to a dielectric conductivity and may be given as

$$\kappa''\omega = 4\pi\sigma . \tag{B12}$$

with the conductivity σ being expressed in esu (statmho-cm/cm^2).

Considering the equality

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \tag{B13}$$

to hold, and substituting Eqs. (B10), (B11), and (B12) into Eq. (B3), leads to the result that for a nonmagnetic medium,

$$\gamma = \frac{j\omega}{c} \sqrt{\kappa - j\,\frac{2\sigma}{f}} = A + jB . \tag{B14}$$

Solving for A and B,

$$\left.\begin{aligned} A &= \frac{2\pi\sqrt{f\sigma}}{c}\sqrt{\sqrt{\left(\frac{\kappa f}{2\sigma}\right)^2 + 1} - \frac{\kappa f}{2\sigma}} \\ \text{and}\quad B &= \frac{2\pi\sqrt{f\sigma}}{c}\sqrt{\sqrt{\left(\frac{\kappa f}{2\sigma}\right)^2 + 1} + \frac{\kappa f}{2\sigma}} \,. \end{aligned}\right\} \tag{B15}$$

However, it may be shown that

$$\phi\left(\frac{\kappa f}{2\sigma}\right) = \sqrt{\sqrt{\left(\frac{\kappa f}{2\sigma}\right)^2 + 1} - \frac{\kappa f}{2\sigma}} = \frac{1}{\sqrt{\sqrt{\left(\frac{\kappa f}{2\sigma}\right)^2 + 1} + \frac{\kappa f}{2\sigma}}} \,. \tag{B16}$$

Therefore

$$\left.\begin{aligned} A &= \frac{2\pi\sqrt{f\sigma}}{c}\,\phi\left(\frac{\kappa f}{2\sigma}\right), \\ \text{and}\quad B &= \frac{2\pi\sqrt{f\sigma}}{c\,\phi\left(\frac{\kappa f}{2\sigma}\right)}; \\ \text{thus}\quad A &= B\phi^2 \,. \end{aligned}\right\} \tag{B17}$$

Substituting Eqs. (B17) into Eqs. (B6), (B7), and (B9) thus gives, in terms of the basic independent parameters, three very useful quantities for describing the plane wave in sea water, i.e., the attenuation rate with depth (or distance), the wavelength in the medium, and the phase velocity:

$$\left.\begin{aligned} \alpha &= 20\left(\log_{10} e\right)\frac{2\pi\sqrt{f\sigma}}{c}\,\phi\left(\frac{\kappa f}{2\sigma}\right), \\ \lambda &= \frac{c}{\sqrt{f\sigma}}\,\phi\left(\frac{\kappa f}{2\sigma}\right), \\ \text{and}\quad v &= c\sqrt{\frac{f}{\sigma}}\,\phi\left(\frac{\kappa f}{2\sigma}\right). \end{aligned}\right\} \tag{B18}$$

Figure B1 shows a plot of the function ϕ defined in Eq. (B16). It is apparent that for sufficiently low values of the argument (which is sometimes referred to as the loss tangent), the factor may be taken as being essentially unity with negligible error. Hence the expressions for α, λ, and v are often approximated for the case of sea water at vlf and below by neglecting this factor, which is equivalent to neglecting κ, the dielectric constant. Figure B1 can be used as a universal curve for comparing the actual values (unstarred) with approximated values (starred), since

$$\phi\left(\frac{\kappa f}{2\sigma}\right) = \frac{\alpha}{\alpha^*} = \frac{\lambda}{\lambda^*} = \frac{v}{v^*} \,. \tag{B19}$$

The approximate values are shown as solid-line curves and the actual values as dashed-line curves in Fig. B2 as functions of frequency for the case of sea water ($\sigma = 3.6 \times 10^{10}$ statmho-cm/cm^2 in esu, 4 mho-m/m^2 in mks units, and $\kappa = 81$ everywhere except near

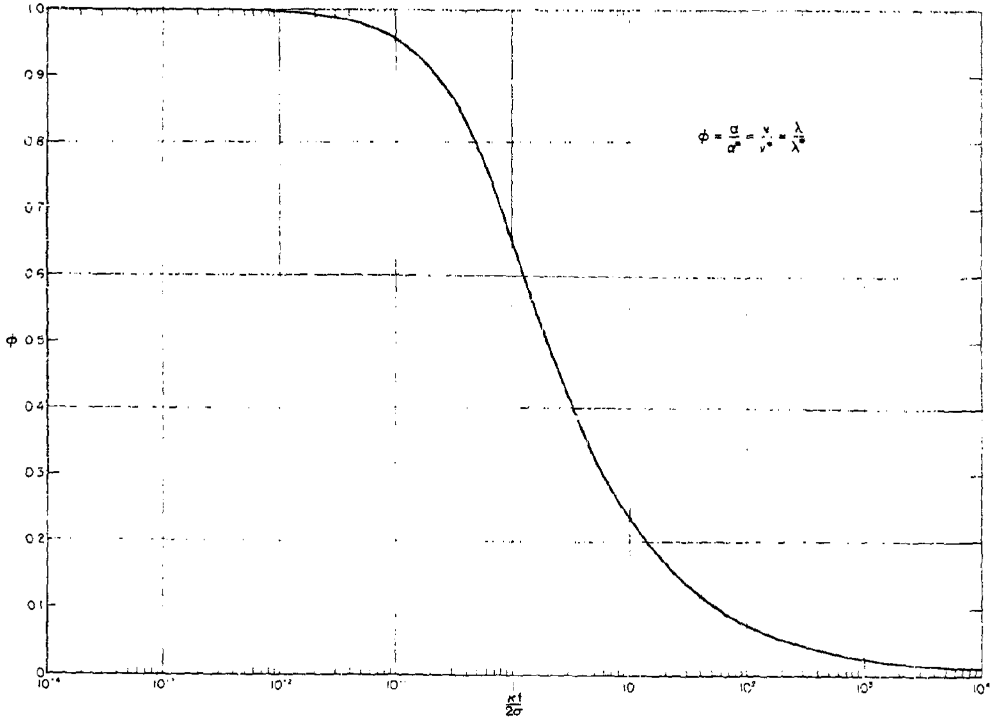
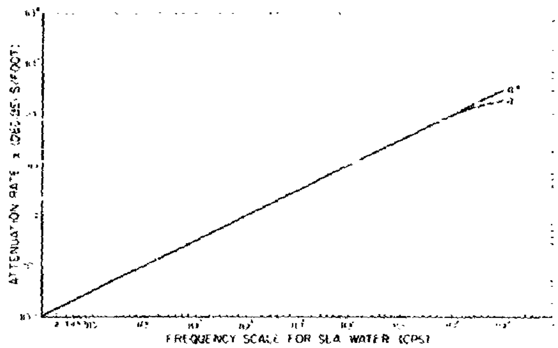
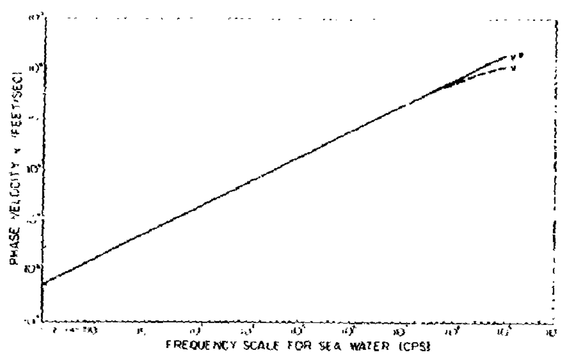


Fig. B1 - *Universal curve showing the effect of the dielectric constant on the rate of attenuation with depth, phase velocity, and wavelength of plane electromagnetic waves in nonmagnetic conductive media.* α, v, λ - Computed values of attenuation, phase velocity, and wavelength when effect of dielectric constant κ is included; α^*, v^*, λ^* - the same as above, but with dielectric constant ignored (i.e., $\kappa = 0$); σ is conductivity in esu, f is frequency in cps.

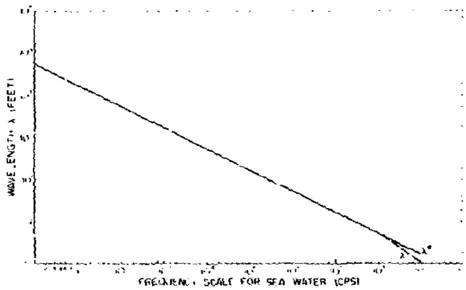


(a) The rate of attenuation α of electromagnetic waves in sea water as a function of frequency



(b) The phase velocity v of electromagnetic waves in sea water as a function of frequency

Fig. B2 - The rate of attenuation with depth, phase velocity, and wavelength of plane electromagnetic waves in sea water as a function of frequency. Starred (*) values are computed with the effect of dielectric constant ignored; i.e., $\kappa = 0$. Unstarred values are computed with the effect of dielectric constant included.



(c) The wavelength λ of electromagnetic waves in sea water as a function of frequency

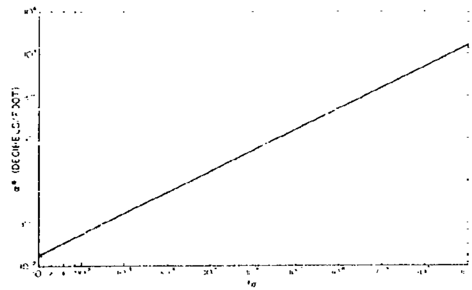
Fig. B2 (Continued) - The rate of attenuation with depth, phase velocity, and wavelength of plane electromagnetic waves in sea water as a function of frequency. Starred (*) values are computed with the effect of dielectric constant ignored; i.e., $\kappa = 0$. Unstarred values are computed with the effect of dielectric constant included.

1000 Mc, where it apparently begins to decrease to about 80.5 at 1000 Mc). Dorsey[2] has made a survey of the literature with regard to the variations to be expected in these parameters in water; however, there appears to be little known about any possible frequency dependence of water conductivity. A convenient plot shown in Fig. B3 is useful for determining the approximate (starred) values for nonmagnetic media other than sea water.[3] The actual values for the other media can then be obtained using Fig. B1, i.e., $\alpha = \alpha^*$ (from Fig. B3) times Φ (from Fig. B1).

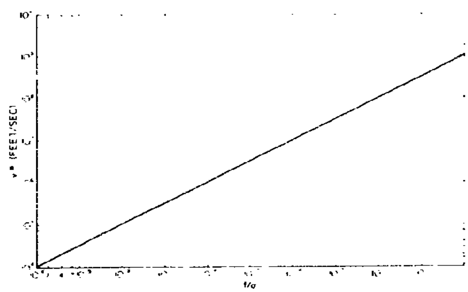
Figure B4 gives a detailed picture of the situation for water with respect to a rather broad range of conductivity and frequency.

[2] N. E. Dorsey, "Properties of Ordinary Water-Substance. . . ," New York:Reinhold, 1940.

[3] Actually, water is listed by Dorsey as being slightly diamagnetic; however, the effect is considered negligible for the purposes of this report.

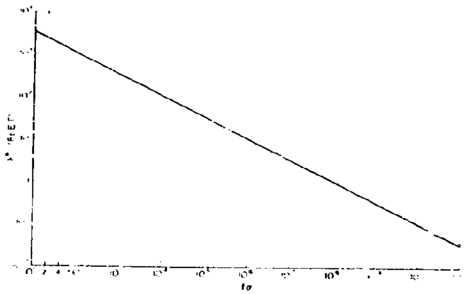


(a) Approximate rate of attenuation α^*



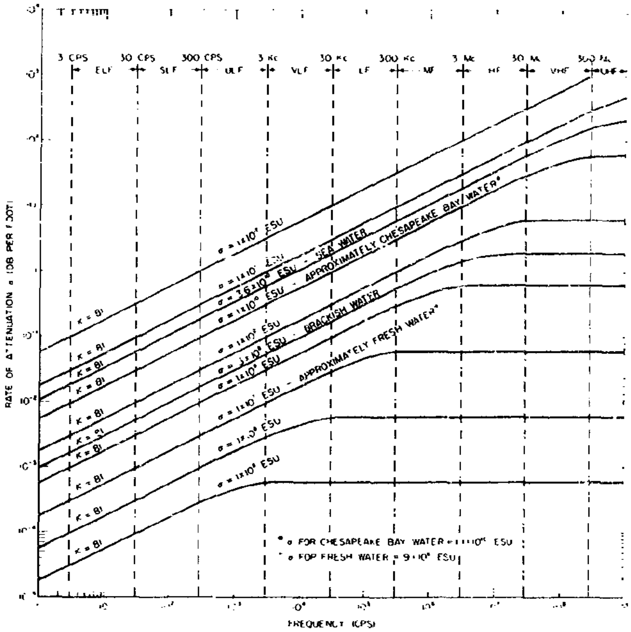
(b) Approximate phase velocity v^*

Fig. B3 - Universal curves for the approximate rate of attenuation with depth, phase velocity, and wavelength of plane electromagnetic waves in nonmagnetic conductive media which neglect the effects of the dielectric constant κ



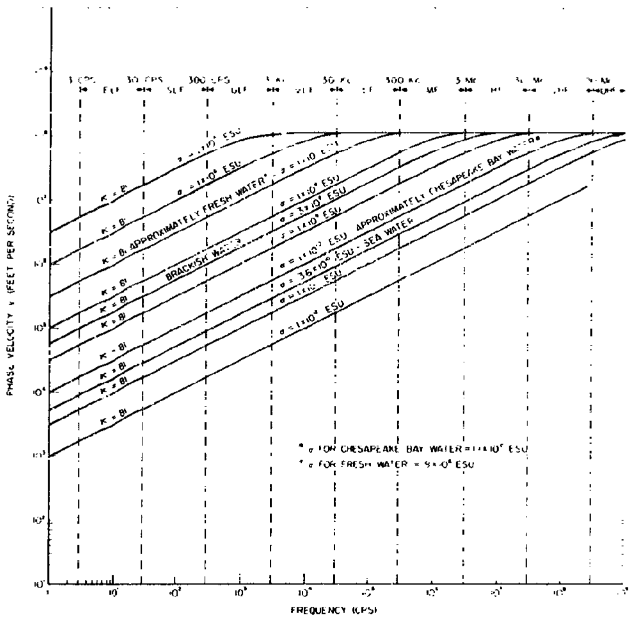
(c) Approximate wavelength λ^*

Fig. B3 (Continued) - Universal curves for the approximate rate of attenuation with depth, phase velocity, and wavelength of plane electromagnetic waves in nonmagnetic conductive media which neglect the effects of the dielectric constant κ



(a) The rate of attenuation α of the underwater electric field with depth as a function of frequency. α in db/ft $= 0.555 \times 10^{-9} \sqrt{f\sigma} \times \sqrt{\sqrt{x^2+1} - x}$. $L_d = \alpha d$ is depth-of-submergence loss in db, and d is depth of loop submergence measured from surface of water to top of loop in feet.

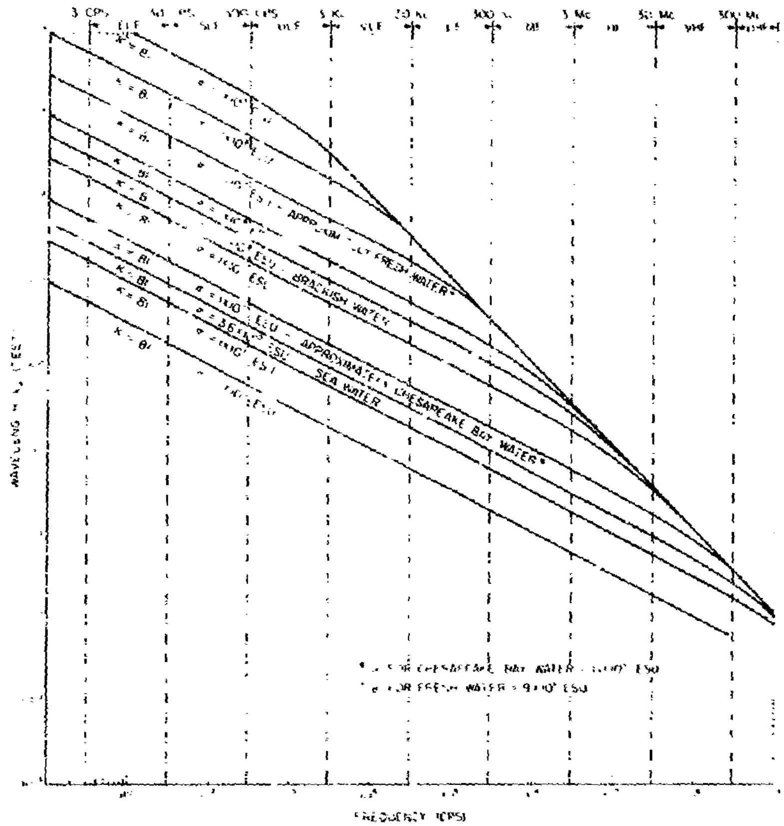
Fig. B4 (Continued) - The rate of attenuation with depth, phase velocity, and wavelength of the radio electric field in water for a wide range of conductivity and frequency. $x = \kappa f/2\sigma$, $\kappa = 81$, $c = 9.835 \times 10^8$ ft/sec, f is in cps, and σ is in esu (statmho-cm/cm^2). For conversion to mks units, σ in esu/(9×10^9) = mho-m/m^2.



(b) The phase velocity v_w of radio waves in water as a function of frequency

$$v_w \text{ in feet per second} = \lambda f = c : \overline{f} \times \sqrt{\sqrt{x^2+1} - x}$$

Fig. B4 (Continued) - The rate of attenuation with depth, phase velocity, and wavelength of the radio electric field in water for a wide range of conductivity and frequency. $x = \kappa f / 2\sigma$, $\kappa = 81$, $c = 9.835 \times 10^8$ ft/sec, f is in cps, and σ is in esu (statmho-cm/cm^2). For conversion to mks units, σ in esu/(9×10^9) = mho-m/m^2.



(c) The wavelength λ_w of radio waves in water as a function of frequency

$$\lambda_w \text{ in feet} = \left(\frac{c}{f}\right) \cdot \sqrt{\sqrt{x^2+1} - x}$$

Fig. B4 (Continued) - The rate of attenuation with depth, phase velocity, and wavelength of the radio electric field in water for a wide range of conductivity and frequency. x = ..., = 81, c = 9.835 × 10^8 ft/sec, f is in cps, and σ is in esu (statmho-cm/cm^2). For conversion to mks units, σ in esu/(9 × 10^9) = mho-m/m^2.

APPENDIX C

DERIVATION OF LOOP-ANTENNA COLLECTION CAPABILITY IN AIR

The difference in potential, or the open-circuit voltage, across the terminals of a loop antenna is equal to the vector sum of the voltages induced in the various parts of the winding. The difference in potential may be represented as a line integral of the "dot product" of the instantaneous electric-field vector with respect to the vector differential element of length over the entire length of the loop. Applying this principle to the case of a rectangular loop in air oriented so that the plane of the loop is parallel to the direction of propagation and the polarization of the electric vector, as indicated in Fig. C1, gives

$$V = n\int_1^5 \mathbf{E}\cdot d\mathbf{S} = n\int_1^2 \mathbf{E}\cdot d\mathbf{S} + n\int_2^3 \mathbf{E}\cdot d\mathbf{S} + n\int_3^4 \mathbf{E}\cdot d\mathbf{S} + n\int_4^5 \mathbf{E}\cdot d\mathbf{S}\,, \tag{C1}$$

or

$$V = n\left[\int_4^5 \mathbf{E}_1\cdot d\mathbf{S} + \int_2^3 \mathbf{E}_2\cdot d\mathbf{S}\right], \tag{C2}$$

since

$$\int_1^2 \mathbf{E}\cdot d\mathbf{S} = \int_3^4 \mathbf{E}\cdot d\mathbf{S} = 0\,. \tag{C3}$$

Therefore, if $a \ll \lambda$,

$$V = n\,b\,k\,(E_1 - E_2)\,. \tag{C4}$$

For sinusoidally varying fields,

$$E_1 = E\cos\omega t\,, \tag{C5}$$

and

$$E_2 = E\cos\left(\omega t + 2\pi\frac{a}{\lambda}\right) = E\cos\left(\omega t + \frac{\omega a}{c}\right). \tag{C6}$$

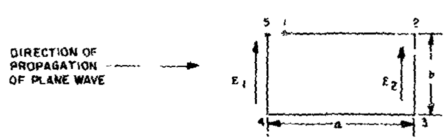


Fig. C1 - Diagram of rectangular loop antenna in air

assuming a negligible decrease in field strength from one side of the loop to the other, but allowing for the change in phase. Thus

$$\frac{E_1 - E_2}{E} = \cos \omega t - \cos\left(\omega t + \frac{\omega a}{c}\right),$$

$$= \cos \omega t - \cos \omega t \cos \frac{\omega a}{c} + \sin \omega t \sin \frac{\omega a}{c}, \tag{C7}$$

$$= \left(1 - \cos \frac{\omega a}{c}\right) \cos \omega t + \sin \frac{\omega a}{c} \sin \omega t .$$

Since the induced voltage will be sinusoidal, being proportional to the difference of two sine waves with the same frequency, Eq. (C7) simplifies to

$$\frac{E_1 - E_2}{E} = A \sin(\omega t + B), \tag{C8}$$

where

$$A = \sqrt{\left(1 - \cos \frac{\omega a}{c}\right)^2 + \sin^2 \frac{\omega a}{c}} = \sqrt{2\left(1 - \cos \frac{\omega a}{c}\right)},$$

and

$$B = \arc\tan\left(\frac{1 - \cos \frac{\omega a}{c}}{\sin \frac{\omega a}{c}}\right).$$

Therefore, substituting this result into Eq. (C4) yields the expression used for the loop-antenna collection efficiency for in-air operation, viz.,

$$\frac{V_A}{E_A} = n b k \sqrt{2\left(1 - \cos \frac{\omega a}{c}\right)}. \tag{C9}$$

Expanding the cosine term under the radical in terms of an infinite power series gives

$$\frac{V_A}{E_A} = n b k \sqrt{2\left[1 - \left(1 - \frac{\omega^2 a^2}{2c^2} + \frac{\omega^4 a^4}{24c^4} - + \cdots\right)\right]}$$

$$= n b k \sqrt{\frac{\omega^2 a^2}{c^2} - \frac{\omega^4 a^4}{12c^4} + - \cdots\cdots}$$

$$= \frac{n \omega a b k}{c} \sqrt{1 - \frac{\omega^2 a^2}{12c^2} + \frac{\omega^4 a^4}{360c^4} - \cdots} . \tag{C10}$$

Therefore, to a very good approximation,

$$\frac{V_A}{E_A} = \frac{n \omega a b k}{c}, \tag{C11}$$

if

$$\frac{\omega^2 a^2}{12c^2} \ll 1 \;(\text{or } fa \ll 5.4 \times 10^8).$$

APPENDIX D

DERIVATION OF LOOP-ANTENNA COLLECTION CAPABILITY IN WATER[1]

The top side of a rectangular loop antenna is considered to be submerged to a depth d in water. The electric field strength just below the surface is taken as $E_w(0)$ and, for the purpose of this derivation, the direction of the electric vector will be considered as lying in a horizontal plane propagating downward (Fig. 1). Applying Eqs. (B2) and (B3) to the case at hand leads to the statement that

$$\left.\begin{aligned} \mathbf{E}_w(d,t) &= \mathbf{E}_w(0,t)\,\epsilon^{j\left(\omega t - \frac{2\pi s}{\lambda}\right) - \gamma d} \\ &= \mathbf{E}_w(0,t)\,\epsilon^{j\left(\omega t - \frac{\omega s}{c}\right) - (A+jB)d} \\ &= \mathbf{E}_w(0,t)\,\epsilon^{-Ad}\,\epsilon^{j\left(\omega t - \frac{\omega s}{c} - Bd\right)}. \end{aligned}\right\} \tag{D1}$$

The field at a depth $(d + b)$ at the bottom of the loop (since the height dimension of the loop is taken as b) is then

$$\left.\begin{aligned} \mathbf{E}_w(d+b,t) &= \mathbf{E}_w(0,t)\,\epsilon^{-A(d+b)}\,\epsilon^{j\left[\omega t - \frac{\omega s}{c} - B(d+b)\right]} \\ &= \mathbf{E}_w(0,t)\,\epsilon^{-Ad}\,\epsilon^{j\left(\omega t - \frac{\omega s}{c} - Bd\right)}\,\epsilon^{-\gamma b} \\ &= \mathbf{E}_w(d,t)\,\epsilon^{-(A+jB)b}. \end{aligned}\right\} \tag{D2}$$

Considering only the scalar parts, since the vectors are parallel, the induced voltage per turn in the loop is taken as

$$\frac{V_w}{n} = \int_0^C \left[E_w(d,t) - E_w(d+b,t)\right] ds. \tag{D3}$$

Subtracting Eq. (D2) from Eq. (D1) then gives

$$E_w(d,t) - E_w(d+b,t) = E_w(0,t)\,\epsilon^{-Ad}\left[1 - \epsilon^{-(A+jB)b}\right]\epsilon^{j\left(\omega t - Bd - \frac{\omega s}{c}\right)}. \tag{D4}$$

Now, substituting Eq. (D4) into Eq. (D3) and performing the integration yields

[1] The development is in general similar to that shown by Norgorden (see Ref. 6, in text), except that here the treatment has been extended and modified to remove certain limiting restrictions.

$$\begin{aligned}\frac{V_W(d,t)}{n} &= E_W(0,t)\ \epsilon^{-Ad}\left[1-\epsilon^{-(A+jB)b}\right]\left[\frac{c}{\omega}\left(\epsilon^{j(\omega t-Bd)}-\epsilon^{j\left(\omega t-Bd-\frac{\omega a}{c}\right)}\right)\right] \\ &= \frac{c}{\omega}\ E_W(0,t)\left[1-\epsilon^{-(A+jB)b}\right]\left[1-\epsilon^{-j\frac{\omega a}{c}}\right]\left[\cos(\omega t-Bd)+j\sin(\omega t-Bd)\right].\end{aligned} \tag{D5}$$

Converting further to trigonometric terminology and taking only the real part gives

$$\frac{V_W(d,t)}{n} = \frac{c}{\omega}\ E_W(0,t)\ \epsilon^{-Ad}\left[p\cos(\omega t-Bd)+q\sin(\omega t-Bd)\right] \tag{D6}$$

where

$$p = \epsilon^{-Ab}\sin Bb\sin\frac{\omega a}{c}+\left(1-\epsilon^{-Ab}\cos Bb\right)\left(1-\cos\frac{\omega a}{c}\right)$$

and

$$q = \epsilon^{-Ab}\left(\sin Bb\cos\frac{\omega a}{c}+\cos Bb\sin\frac{\omega a}{c}-\sin Bb\right)-\sin\frac{\omega a}{c}.$$ [2]

Equation (D6) may now be used to establish the loop pickup capability in water (the peak magnitude of the induced voltage for a given peak magnitude of electric field at the top of the loop) by taking the square root of the sum of the squares of the sine and cosine terms. Hence

$$\frac{V_W}{E_W} = \frac{V_W(d)}{E_W(d)} = \frac{nc}{\omega}\sqrt{p^2+q^2}\,. \tag{D7}$$

where the peak magnitude of the electric field at the top of the loop is

$$E_W(d) = E_W(0)\ \epsilon^{-Ad}\,. \tag{D8}$$

Considerable simplification results whenever the horizontal loop dimension a is small enough and the frequency is low enough so that

$$\frac{\omega a}{c} \ll 1,\qquad \cos\frac{\omega a}{c} \doteq 1$$

and

$$\sin\frac{\omega a}{c} \doteq \frac{\omega a}{c}\,.$$

for then Eq. (D7) may be approximated as

$$\frac{V_W}{E_W} = n a k\sqrt{1-2\epsilon^{\theta}\cos\frac{\theta}{\phi^2}+\epsilon^{2\theta}}\,. \tag{D9}$$

---

[2]Using the notation later introduced in Eq. (D9),

$$p = \epsilon^{\theta}\sin\frac{\theta}{\phi^2}\sin\frac{\omega a}{c}+\left(1-\epsilon^{\theta}\cos\frac{\theta}{\phi^2}\right)\left(1-\cos\frac{\omega a}{c}\right)$$

and

$$q = \epsilon^{\theta}\left[\sin\left(\frac{\theta}{\phi^2}+\frac{\omega a}{c}\right)-\sin\frac{\theta}{\phi^2}\right]-\sin\frac{\omega a}{c}\,.$$

where

$$[illegible] = -Ab = \frac{-2 [illegible] b [illegible]}{c} [illegible] \overline{f [illegible]} ,$$

$$\frac{[illegible]}{[illegible]^2} = Bb = -\frac{2 [illegible] b}{c [illegible]} \overline{f [illegible]} .$$

and k is introduced to allow for different types of units (in this report k is taken as 0.3048006 meters/ft).

Furthermore, if

$$\frac{\kappa f}{2[illegible]} << 1, \qquad [illegible] \doteq 1 ,$$

then

$$\frac{V_W}{E_W} = n a k \sqrt{1 - 2\epsilon^{\theta^*} \cos [illegible]^* + \epsilon^{2\theta^*}} , \tag{D10}$$

where

$$\frac{[illegible] a}{c} << 1 ,$$

and, as in Eq. (B19), [illegible][3]

---

[3] Equation (D10) is essentially equivalent to an expression derived by Norgorden (see p. 6, Ref. 6, in text).

APPENDIX E

DERIVATION OF THE RECTANGULAR LOOP HEIGHT CORRESPONDING TO THE MAXIMUM ANTENNA COLLECTION CAPABILITY IN WATER

If the collection capability is maximum with respect to loop height, then its derivative with respect to loop height must be zero; hence, taking the derivative of Eq. (13) in the text with respect to b and setting it equal to zero gives

$$0 = \frac{d\left(\frac{V_w}{E_w}\right)}{db} = \frac{2\pi a k \, e^{\theta}(e^{\theta} - \cos\theta + \sin\theta)}{b\sqrt{1 - 2e^{\theta}\cos\theta + e^{2\theta}}} \, . \qquad \text{(E1)}$$

or after eliminating nonpertinent factors,

$$0 = e^{\theta} - \cos\theta + \sin\theta \, . \qquad \text{(E2)}$$

This transcendental equation has been solved for the particular root of interest, using the Newton numerical iteration technique, giving θ = -2.284102297 . . . , or (solving θ for b) for $\sigma/f > 3600$,

$$b = \frac{2.284102297 \, c}{2\pi \sqrt{f\sigma}}$$

$$= \frac{3.57553524 \times 10^{8}}{\sqrt{f\sigma}}$$

$$= 0.36352617 \, \lambda_w \, . \qquad \text{(E3)}$$

since $\lambda_w = c/\sqrt{f\sigma}$.

APPENDIX F

DERIVATION OF FREQUENCY FOR MAXIMUM V_W/E_A FOR A GIVEN DEPTH OF OPERATION AND SIZE OF LOOP ANTENNA

Setting the derivative of V_W/E_A with respect to frequency equal to zero serves to establish the conditions necessary for achieving a maximum V_W/E_A at a given depth of operation for a given size of loop antenna. This is done in Eq. (F1) and is then simplified by eliminating nonpertinent factors. Thus

$$0 = \frac{d\left(\frac{V_W}{E_A}\right)}{df} = \frac{-\mu n \pi b k \cdot \frac{d}{b}(\epsilon^{\theta} + \sin\theta - \cos\theta)}{2\,\sigma\,\sqrt{1 - 2\epsilon^{\theta}\cos\theta + \epsilon^{2\theta}}} \left[\theta\epsilon^{\theta} + \frac{\left(1 + \theta\frac{d}{b}\right)(1 - 2\epsilon^{\theta}\cos\theta + \epsilon^{2\theta})}{\epsilon^{\theta} + \sin\theta - \cos\theta}\right], \tag{F1}$$

or

$$0 = \theta\epsilon^{\theta} + \frac{\left(1 + \theta\frac{d}{b}\right)(1 - 2\epsilon^{\theta}\cos\theta + \epsilon^{2\theta})}{\epsilon^{\theta} + \sin\theta - \cos\theta}. \tag{F2}$$

The particular frequency f_0 corresponding to a maximum induced voltage in a submerged loop for a given field strength in air can now be found by solving for the value of θ which will satisfy Eq. (F2) (the particular value obtained being θ_0, which is, of course, dependent on the ratio of d/b) and substituting the value obtained for θ_0 into the following equation, which is merely a rearrangement of the original equation defining θ (valid whenever $\sigma/f > 3600$):

$$f_0 = \frac{1}{\sigma}\left(\frac{\theta_0\, c}{2\pi\, b}\right)^2. \tag{F3}$$

Solutions to the transcendental Eq. (F2) have been obtained using iterative numerical techniques. Figure F1 is a plot of the values obtained for the magnitude of θ_0 as a function of d/b. Figure F2 has been plotted to facilitate the determination of the optimum frequency f_0 and is simply a graphical plot of Eq. (F3). Thus, given the values of d and b, the magnitude of θ_0 may be obtained from Fig. F1, and then having $|\theta_0|$ and knowing the value of σ, f_0 can be read directly from Fig. F2 (provided, of course, that the ratio of σ to f_0 is greater than 3600).

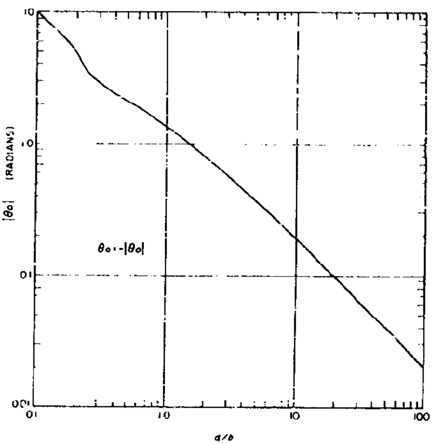


Fig. F1 - A plot of the root of Eq. (F2) as a function of d/b

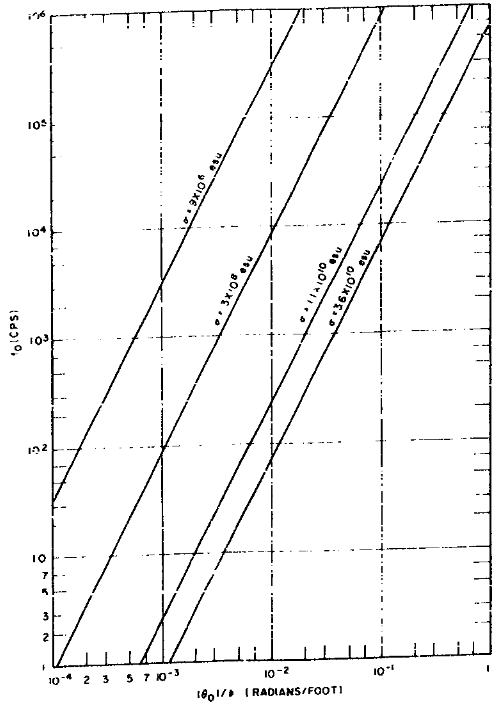


Fig. F2 - The particular frequency f_0 which gives maximum induced voltage in a submerged loop antenna with a given fixed field strength in air as a function of $|\theta_0|/b$ and certain values of σ

LIST OF SYMBOLS AND THEIR DEFINITIONS

α Attenuation per unit of increasing depth in water, $\alpha = 0.555\times10^{-7} \times \sqrt{f\sigma} \times \sqrt{\sqrt{\left(\frac{\kappa f}{2\sigma}\right)^2 + 1} - \frac{\kappa f}{2\sigma}}$, in decibels per foot

α^* Approximate value of α obtained by neglect of dielectric constant κ: $\alpha^* = 0.555\times10^{-7}\sqrt{f\sigma}$, in decibels per foot

a Length of rectangular loop antenna, in feet

A Attenuation or amplitude factor

b Height dimension of rectangular loop antenna, in feet

B Phase factor

γ Complex propagation factor

c Velocity of light in air, $c = 9.83570\times10^8$ feet per second

d Depth of loop antenna submergence in water, measured from water surface to top of loop, in feet

D Range or distance in air between transmitting and receiving antennas, in nautical miles

dbv $20 \log_{10} (v_1/v_2)$(see footnote 9)

dbr $10 \log_{10} (r_1/r_2)$(see footnote 9)

$d\mathbf{s}$ Vector differential element of length s

ϵ Base of natural (Napierian) logarithms, $\epsilon = 2.7182818 \ldots$

ϵ_0 Permittivity of free space

ϵ^* Complex permittivity

ϵ' Real part of ϵ^*

ϵ'' Imaginary part of ϵ^*

E_A Magnitude of radio electric field in air at water surface, in volts per meter (v/m)

E_{A_0} Receiving-system threshold field sensitivity, i.e., the magnitude of radio electric field in air at the water surface at range D which will provide design threshold sensitivity conditions with in-air operation, in volts per meter. (Note: with surfaced operation, E_{A_0} is equal to E_0.)

E_0 Magnitude of radio electric field in air at water surface at range D for minimum satisfactory receiving-system output performance, in volts per meter

E_s The field intensity required under actual operational system conditions for the same signal-to-noise ratio and performance conditions outlined for E_0, i.e., E_s E_0/l_s, in volts per meter

$E_w(d)$ Magnitude of radio electric field in water at depth d, in volts per meter

$E_w(0)$ Magnitude of radio electric field in water at zero depth of loop submergence, i.e., top of loop at water surface, in volts per meter

$\mathbf{E}$ Vector quantity representing the electric field intensity

$\mathbf{E}_1$ Instantaneous electric vector field at the vertical side of the rectangular loop antenna closest to the transmitter

$\mathbf{E}_2$ Instantaneous electric vector field at the vertical side of the rectangular loop antenna furthest from the transmitter (taken at same instant of time as for $\mathbf{E}_1$)

E_1 Scalar magnitude of $\mathbf{E}_1$

E_2 Scalar magnitude of $\mathbf{E}_2$

$\mathbf{E}_0$ Initial value of $\mathbf{E}$

$\mathbf{E}_w(d,t)$ Vector quantity representing instantaneous electric field intensity in water

$\mathbf{E}_w(0,t)$ $\mathbf{E}_w(d,t)$ for $d = 0$

$E_w(0,t)$ Scalar magnitude of $\mathbf{E}_w(0,t)$

f Transmission or signal frequency, in cps unless otherwise specified

f_0 Frequency which gives maximum induced voltage in a submerged-loop antenna with any particular given <u>fixed</u> field strength in air, in cps

F_{CA} Relative rectangular open-core loop-antenna collection capability in air (referred to the reference loop R) which incorporates effects due to number of turns and loop dimensional changes (expressed as a ratio, or given in db)

F_{CW} Relative rectangular open-core loop-antenna collection capability in water (referred to the reference loop R) which incorporates effects due to number of turns and loop dimensional changes (expressed as a ratio, or given in db)

F_{VIL} Relative rectangular open-core loop antenna voltage-interface loss (referred to that of a reference loop R) which incorporates effects due to loop dimensional changes (expressed as a ratio)

G Loop-antenna pickup capability gain or improvement with operation in water as compared to in-air operation

h Antenna effective height, in feet

$\mathbf{H}$ Vector quantity representing the magnetic field intensity

$\mathbf{H}_0$ Initial value of $\mathbf{H}$

[illegible]	A parameter used for notational convenience; [illegible] $= \frac{-2\pi b d}{\lambda}\sqrt{f\sigma}$ [illegible] in radians
$[illegible]_0$	Value of [illegible] (or the root) which satisfies Eq. (F2)
[illegible]*	Approximate value of [illegible] obtained by neglect of dielectric constant [illegible]
I	Antenna down-lead current, in amperes
k	A conversion factor, $k = 0.3048006$ meters per foot
κ^*	Complex relative permittivity
κ'	The real part of κ^* (also referred to as κ, the dielectric constant)
κ''	Imaginary part of κ^*
κ_m^*	Complex relative permeability
κ_m'	The real part of κ_m^*
κ_m''	Imaginary part of κ_m^*
l_s	System operational loss; $l_s = E_0/E_s$
L_d	Loss due to loop-antenna submergence to a specified depth in water $L_d = \alpha d = -20 \log_{10} E_W(d)/E_W(0)$, in decibels
L_s	Allowed system operational loss; $L_s = -20 \log_{10} l_s$, in decibels
L_v	Voltage-interface loss; $L_v = -20 \log_{10} (V_W(0)/V_A)$
λ	The radio wavelength in air, in feet
λ_W	The radio wavelength in the conducting medium (water), in feet
λ^*	Approximate value of λ obtained by neglect of dielectric constant [illegible]
μ_0	Permeability of free space
μ^*	Complex permeability
μ'	Real part of μ^*
μ''	Imaginary part of μ^*
n	Number of turns on loop antenna
$P_{1,2}$	Power applied to resistances, $r_{1,2}$
P_r	Transmitter radiated power, in watts
p, q	Parameters used for notational convenience, defined in Eq. (D6)
$r_{1,2}$	Resistances

R Reference, single-turn, single-plane, one-foot-square, open-core loop antenna (often employed as subscript to other symbols to designate the reference-loop case)

R_r Antenna radiation resistance the value of resistance in series with the antenna down-lead current which results in the antenna radiated power, i.e., $R_R = P_r/I^2$, in ohms

σ Electrical conductivity of water, in esu (statmho-cm/cm^2)

s Distance variable of integration, taken along horizontal of rectangular loop with origin at side nearest the source of electromagnetic radiation

S_f Surplus radio field, $20 \log_{10}(E_A/E_{A_0})$

S_0 Design threshold cw sensitivity, i.e., that unmodulated signal input necessary for a receiver output signal-to-noise ratio $(S/N)_0$ equal to 0 decibels at the standard 6-mw output power level

S_{20} Standard cw sensitivity, i.e., that unmodulated signal input necessary for a receiver output signal-to-noise ratio $(S/N)_{20}$ equal to 20 decibels at the standard 6-mw output power level

T Time period

t Time, in seconds

v Phase velocity of propagation

v^* Approximate value of v obtained by neglect of the dielectric constant κ

V Instantaneous voltage induced in a loop antenna, in volts

V_A Voltage induced in loop antenna in air by radio field E_A, in volts

V_0 Design threshold voltage sensitivity, i.e., the voltage induced in a loop antenna in air by a radio field E_{A_0}, in volts

$V_W(0)$ Voltage induced in loop antenna in water at zero depth of submergence, i.e., with top of loop at water surface, by radio field in air E_A, in volts

$V_W(d)$ Voltage induced in loop antenna in water at a specified depth of loop submergence by a radio field in air E_A, in volts

$V_W(d, t)$ Instantaneous voltage induced in loop antenna in water at a specified loop depth

$(V_A/E_A)_0$, $(V_W/E_W)_0$, $(V_W/E_A)_0$ Convenient arbitrary references of induced or open-circuit terminal voltages and reference fields, all being 1 volt and a field of 1 volt per meter (see detailed discussion on p. 34)

τ A parameter used for convenience, $\tau = \sqrt{\sqrt{\left(\frac{\kappa f}{2\sigma}\right)^2 + 1} - \frac{\kappa f}{2\sigma}}$

X Distance

UNITED STATES GOVERNMENT

memorandum

5300-31

DATE: 29 June 1998

REPLY TO ATTN OF: Code 5300

SUBJECT: REQUEST TO DECLASSIFY NRL REPORTS

TO: Code 1221.1 (C. Rogers)

1. It is requested that the NRL Reports listed below be declassified. The information contained in these reports has become public knowledge in the many years since first classified.

Declassified, public release.

3706 AD-C954460	4371 AD-C38472 ✓	[redacted]	[redacted]	5338 AD-312117 ✓	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	5399 AD-314113 ✓	5540 AD-320263 ✓
[redacted]	[redacted]	5126 AD/62728 ✓	5247 AD-304847 ✓	5403 AD-312699 ✓	5570 AD-320955 ✓
[redacted]	[redacted]	[redacted]	[redacted]	5441 AD-315354 ✓	
4284 AD031076 ✓	4878 AD-125352 ✓	5197 AD302457 ✓	[redacted]	5536 AD-343165 ✓	

Declassify, DoD and DoD contractors only.

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TKS
Mary Templeman

G.V. Trunk

G.V. TRUNK
Superintendent
Radar Division

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GSA FPMR (41 CFR) 101-11.6
5010-114